CONTRIBUTIONS FROM THE LICK OBSERVATORY NO. 3.

TERRESTRIAL

ATMOSPHERIC ABSORPTION

OF THE

PHOTOGRAPHIC RAYS OF LIGHT.

BY

J. M. SCHAEBERLE,
Astronomer in the Lick Observatory.

Printed by authority of the Regents of the University of California.

SACRAMENTO:
STATE OFFICE, : : : A. J. JOHNSTON, SUPT. STATE PRINTING.
1893.

ORGANIZATION OF THE LICK OBSERVATORY.

HON. T. G. PHELPS, HON. C. F. CROCKER, HON. H. S. FOOTE,

Committee of the Regents on the Lick Observatory.

MARTIN KELLOGG, ... *President of the University.*

EDWARD S. HOLDEN, ... *Director and Astronomer.*

J. M. SCHAEBERLE, ... *Astronomer.*

E. E. BARNARD, ... *Astronomer.*

W. W. CAMPBELL, ... *Astronomer.*

ALLEN L. COLTON, ... *Assistant Astronomer.*

C. D. PERRINE, ... *Secretary.*

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TERRESTRIAL ATMOSPHERIC ABSORPTION OF THE PHOTOGRAPHIC RAYS OF LIGHT.

By J. M. Schaeberle.

The remarkable revolution in the methods of charting celestial configurations, brought about by substituting for the photographic plate, the human eye, has opened up a most inviting field of investigation. To obtain results which heretofore demanded months and years of labor on the part of the observer only a few hours are now required.

As a necessary consequence of this radical change in the methods of work many new problems confront the astronomer, some of which must be solved before the information given by the photographs can be presented in its final form.

The human eye as normally constituted is most sensitive to a particular set of light rays. If now we could construct a photographic plate, on which the set of rays which are most effective visually would also be most effective photographically, it is probable that the relative brightness determined photographically would not differ from that deduced from direct visual observation.

Up to the present time, however, the plates which have been universally employed in photographic work are so prepared that the action of the light from near the red end of the solar spectrum—or where the light is most effective visually—is very much less effective than that coming from near the violet end. For this reason it would seem to follow at once, that whatever unit of brightness is chosen, the relation between the visual brightness and that deduced from the action of the same source of light on the photographic plate, can only be considered constant so long as the spectral type remains the same. In general we should expect that for stars of different types of spectra, the relation between the visual magnitudes will not be the same as the relation between the corresponding photographic magnitudes.

Investigations relating to the photographic magnitudes of the fixed stars have been made by Pickering, Pritchard,

CHARLIER, SCHEINER, GOULD, and others; a consideration of the various results seems to show that the different forms of photographic telescopes and plates do not, as a rule, give, under otherwise similar conditions, exactly the same data. As will be shown farther on, the law deduced by the present writer holds good for the three different telescopes available; two being of 6-inch aperture, and the third 33-inch. SEED plates, Sensitometer No. 26, were used in all cases.

This line of work was taken up in 1889, at the suggestion of Professor HOLDEN. An equatorially-mounted DALLMEYER lens, primarily intended for eclipse work at Cayenne, and loaned to the Lick Observatory by the United States Naval Observatory, was first employed for obtaining the necessary data; later on a WILLARD lens, belonging to the CROCKER telescope, was also used.

DETERMINATION OF THE RELATION WHICH, FOR A GIVEN STAR, EXISTS BETWEEN THE APERTURE (Q) OF THE TELESCOPE AND THE DIAMETER (d) OF THE STAR'S IMAGE FOR A GIVEN EXPOSURE TIME (t).

Some of the results of a preliminary investigation made on Mount Hamilton are embodied in a paper entitled, "On the Photographic Magnitudes of the Fixed Stars." (See *Publications of the Astronomical Society of the Pacific*, Vol. I, No. 4.)

To obtain a general expression for the brightness of a fixed star, as determined by means of its image impressed upon the photographic plate during an exposure time t, and with aperture D, I arranged the following scheme for obtaining the necessary data:

With a known aperture of the objective, a series of images of the star were first obtained, the exposure times being respectively 1^s, 2^s, 4^s, 8^s, 16^s, 32^s, 64^s, and 128^s, the telescope being slightly shifted after each exposure to keep the images from overlapping. Other series of similar exposures on the same star and plate were then made with different known apertures. This scheme was then applied to different stars.

Now, in the case of any one of these stars, the source of light, during the time of one series of exposures, remains practically constant; hence, it is evident that the relation between the exposure time t, the diameter of the aperture D, and the

diameter of the star's image d, must always be such that the expression for the brightness B is a constant quantity for any given star, whatever its magnitude may be.

From a discussion of the data given by these plates, I found that the law governing the size and rate of growth of the image could be expressed by means of an equation of the form,*

$$d = \alpha + \beta \log D + \gamma D \log t. \tag{1}$$

In which d is the measured diameter of the image for the aperture D and exposure time t, while α, β, and γ are constants which depend upon the telescope, the atmospheric condition, and the kind of photographic plate employed.

Using this theoretical relation between d, D, and t, I showed that if Q represents the theoretical aperture which a standard star (*Polaris*) would require to produce in the time t, an image having the same diameter d as that produced by any star with a constant aperture Q_0 (6-inch) in the same time t, the equation which serves to determine the magnitude of any star whose image is impressed upon the photographic plate is of the following form:

$$d = \alpha + \beta \log Q + \gamma Q \log t. \tag{2}$$

The corresponding photographic magnitude m' is then found by means of the expression

$$m' = 1 - \frac{\log k Q^2}{\log 0.4} \tag{3}$$

k being a constant which depends upon the photographic magnitude of the standard star. If we take *Polaris* as the standard star, and assume for the present (only) its apparent photographic magnitude to be 2.00 at the zenith-distance 52° 40′, corresponding to the co-latitude of the Lick Observatory, equation (3) becomes

$$m' = 2 - \frac{\log Q^2}{0.4} \tag{4}$$

(For illustrative examples, see *Publications of the Astronomical Society of the Pacific*, Vol. I, No. 4.)

* As I afterwards learned, Professor PRITCHARD, Director of the Savilian Observatory, had previously also found that d could be expressed as a function of log t.

DETERMINATION OF THE INSTRUMENTAL CONSTANTS.

I shall now attempt to show that equation (2), when properly interpreted, is quite general in its character, and apparently applicable to similar telescopes of any aperture actually employed in photographic work.

Let us first consider the case of two similar telescopes having equal apertures. From a series of experiments made with two such instruments (one of which was a DALLMEYER, the other a WILLARD lens of 5.9 inches aperture) I found that while the growth of the images during equal units of time was practically the same for both instruments, there was a small and almost constant difference between the dimensions of the images for the same value of t. This difference, taking the DALLMEYER telescope as the standard, might be called the constant correction of the WILLARD lens.

In order, therefore, to render the measured quantities homogeneous, we must first determine the correction to be applied to each d of one instrument, in order to reduce it to the normal d of a particular instrument taken as a standard.

In the following notation let the symbols which are not primed refer to the standard (6-inch DALLMEYER) telescope, and let those with the primes refer to a companion telescope of equal (6-inch) aperture.

Let d_0 and d_0' denote the diameters corresponding to the exposure time t_0:

Let d and d' denote the diameters corresponding to the exposure time t:

And for brevity let

$$\alpha + \beta \log Q = c \qquad (5)$$
$$\alpha' + \beta \log Q = c' \qquad (6)$$

Then, since the value of Q is the same for both telescopes when the same star is observed, we can write

$$d_0 = c + \gamma\, Q \log t_0 \qquad (7)$$
$$d = c + \gamma\, Q \log t \qquad (8)$$

and

$$d_0' = c' + \gamma'\, Q \log t_0 \qquad (9)$$
$$d' = c' + \gamma'\, Q \log t \qquad (10)$$

from which the expressions for Q become

$$Q = \frac{d - d_0}{\gamma\,(\log t - \log t_0)} \tag{11}$$

$$Q = \frac{d' - d_0'}{\gamma'\,(\log t - \log t_0)} \tag{12}$$

Hence, the difference between the diameters of any two images of the same star divided by the difference between the logarithms of the corresponding times of exposure is a constant for the same telescope and plate.

We must evidently have also

$$\frac{d - d_0}{\gamma} = \frac{d' - d_0'}{\gamma'} \tag{13}$$

Now, although the values of d and d' in the two telescopes may differ considerably for the same value of t, still experiment seems to prove that the difference between the *growths* ($d - d_0$ and $d' - d_0'$) of the images in two similar telescopes can be treated as a quantity of the second order, so that we can write

$$\gamma = \gamma' \tag{14}$$

If now we make $t_0 = 1^s$, the expressions for the value of Q become

$$Q = \frac{d - d_0}{\gamma \log t} \tag{15}$$

$$Q = \frac{d' - d_0'}{\gamma \log t} \tag{16}$$

the value of γ for the Dallmeyer telescope and Seed 26 plates being 0.0033.

It is evident that a difference in the development of the plates may have a great effect upon the resulting values of d. A strong development will, as a rule, give larger images than a weak development, and as the image of a bright star grows faster than that of a faint star the relative effect may be most marked. Therefore, *not only should the plates be of the same degree of sensitiveness, but the development of these plates should be uniformly the same.*

Equations (15) and (16) can be considered as special differentials of (8) and (10), in which the increments are finite. For if we differentiate (8) and (10), regarding d and t as variables, and designating the differential by the symbol δ, we obtain

$$\delta\, d = \gamma\, Q \frac{t}{t} \qquad (17)$$

$$\delta\, d' = \gamma\, Q \frac{\delta\, t}{t} \qquad (18)$$

from which we have

$$Q = \frac{\delta\, d}{\gamma\, \delta \log t} \qquad (19)$$

$$Q = \frac{\delta\, d'}{\gamma\, \delta \log t} \qquad (20)$$

which are identical with (15) and (16) when finite increments are employed.

To determine the value of the constant correction to be applied to the data given by the companion telescope, we first find the value of Q by means of (16). With this Q as an argument we enter Table II, and take out the values of d corresponding to the exposure time t; then since we also have, according to equations (7), (8), or (9), (10), the equation

$$c - c' = d - d' \qquad (21)$$

it follows that each exposure on a given star furnishes an independent value of the correction $(c - c')$ to be applied to the measured values of d' to obtain the normal or tabular values. It also follows that if the empirical formula is correct, the several independent values of these corrections should agree within the limits of the errors of observation.

Thus far we have been considering the problem of determining, with the aid of data given by an assumed standard telescope, the photographic magnitude of a star from the data given by a second telescope having the same aperture as the standard instrument. Let us now consider the general problem of finding the photographic magnitude from data given by a telescope having an aperture $n\, Q_0$ referred as before to the system of magnitudes given by the standard telescope.

Now, for theoretically perfect telescopes, the magnitude M' corresponding to a given d, t, and aperture $n\ Q_o$ can be expressed by means of the equation

$$M = 2 - \frac{\log\left(\frac{Q}{n\ Q_0}\right)^2}{0.4} \tag{22}$$

And for the same values of d and t, but with the aperture Q_o, the required magnitude (m') necessary to satisfy the conditions would be expressed by equation (4). The difference between equations (4) and (22) for the same t and d will evidently be a constant quantity, whose value is given by the equation

$$M' - m' = 5 \log n \tag{23}$$

or,

$$M' = m' + 5 \log n \tag{24}$$

Hence, with the aid of Table II we should also be able to determine the theoretical photographic magnitude of a star photographed with an aperture $n\ Q_o$ by simply adding 5 log n to the tabular m' corresponding to the observed arguments d and t.

However, in deducing the law expressed by equation (1) all the imperfections peculiar to the particular standard instruments are involved; that is, the law is so determined that the constant corrections are already applied. But the imperfections of another telescope will not necessarily be the same as those of the standard instrument, so that generally the measured values of d will be in error when referred to the standard instrument. It is therefore essential to use the corrected value of d'; or to determine the value of Q by means of equation (12) or (16) where the constant is eliminated.

As I very much desired to learn how closely these formulæ represented the observations for those cases in which Q_o and $n\ Q_o$ were very different, the theory was tested by means of the most extreme practical case which could be applied at the present time. At my request Professor HOLDEN, aided by Professor CAMPBELL, made a series of suitable exposures for me upon the star *Polaris* with the great refractor, the clear aperture of which for photographic purposes is 33 inches.

As our unit of aperture is 6.00 inches we have for substitution in equation (22) the values

$$\left.\begin{aligned} Q_\circ &= \tfrac{6}{6} = 1.00 \\ n\,Q_\circ &= \tfrac{33}{6} = 5.50 \end{aligned}\right\} \qquad (25)$$

The true tabular magnitude m' is therefore according to equation (24),

$$M' = m' + 5 \log 5.5 = m' + 3.70.$$

To compare this result with actual observation we employ equation (16) in order to eliminate the constant errors of the d (given in the table) in determining the value of Q.

From the observed data we have for values of $t = 1^s$ and $t = 256^s$, the corresponding values $d' = 0.0250$ and $d' = 0.0700$; hence, according to actual observations we have

$$n\,Q^\circ = 5.67 \qquad (26)$$

Agreeing fairly well with the theoretical value 5.50 found above. The tabular magnitude corresponding to $n\,Q_\circ = 5.67$ is -1.77; hence, according to equation (24),

$$m = 3.70 - 1.77 = 1.93 \qquad (27)$$

When it is considered that this result for the magnitude of *Polaris* (differing only $0^m.07$ from the adopted magnitude) is practically the same as that given by the 6-inch objective, it would seem to indicate that the results obtained with different instruments are less heterogeneous than might naturally be expected, for in the present case not only are the apertures very different, but for the 33-inch telescope the ratio of aperture to focal length is only about one third as great as it is for the DALLMEYER lens. Discrepancies in the results given by different observers are probably largely due to the fact that the constants peculiar to each instrument and plate have not been sufficiently sharply determined, and still more largely due to differences in the degree of development of the photographic plates.

To show the practical agreement between theory, as defined by equation (2), and observation, I give the data obtained with the great telescope, using all the exposures made on *Polaris*.

In the following table the first column gives the duration of the exposures; the second column, the diameters of the corresponding stellar images; the third column, the tabular diameters corresponding to $n\ Q = 5.67$, as found by observation; the last column contains the individual values of the constant correction $c - c_0$ to be applied to all measured values of d' to make them comparable with the values given in Table II.

EXPOSURES ON POLARIS WITH THE 33-INCH PHOTOGRAPHIC TELESCOPE, AND COMPARISON WITH THEORY, FOR TESTING THE LAW DEDUCED FROM EXPERIMENTS MADE WITH A 6-INCH PHOTOGRAPHIC TELESCOPE.

TABLE I.

Exposure Time.	Measured d'	Computed d	Constant $c - c_0$
1^s	0.0250	0.0079	0.0171
2	.0305	.0136	.0169
4	.0360	.0192	.0168
8	.0415	.0248	.0167
16	.0470	.0305	.0165
32	.0525	.0361	.0164
64	.0590	.0417	.0173
128	.0645	.0474	.0171
256	.0700	.0530	.0170

On the hypothesis that the standard and comparison telescopes have, photographically, the same peculiarities, the theoretical value of the constant term for the 33-inch telescope is given by the expression

$$\alpha + \beta \log Q'_0 = 0.0055 + 0.0033 \times 0.74 = 0.0079 \quad (28)$$

Polaris being taken as the standard star. But according to observation the mean constant is 0.0248; the difference between this value and that deduced from equation (2) must be attributed to the peculiarities of the 33-inch lens as compared with the 6-inch standard telescope. The explanation of this difference seems to be that the initial images do not start from a point, but from a sensible area; the magnitude of this area is not only dependent upon the diameter of the objective but also upon the character of the color curve.

The photographic magnitude of any star can best be determined by making two or more exposures on the same plate, so

as to give suitable values of d' for determining the *rate of growth of the image.*

The law expressed by equation (2) will then, it seems, hold good; equations (16) and (24) being used to find the proper magnitude.

In order to apply the formulæ to any particular case, the equivalent value of d should not be much less than 0.0055, as the formulæ could not well be tested, on the standard star, for values of t less than one second of time.

The number of examples could, of course, be multiplied, but for the present purpose the foregoing illustrations are deemed sufficient, so far as the investigation of the atmospheric absorption of the photographic rays of light is concerned. Later on, in dealing with the data given by the WILLARD lens, this subject will be further illustrated.

The foregoing investigations were, of course, necessary before the observations on absorption could be reduced. To enable any one to follow the various steps, and also to make use of the tabular values for other purposes, all the necessary data are given in abbreviated form. In making the exposures the observer was not always able to guard against those causes which produce imperfect images, nor was it always possible to know before the developments of the plates whether such imperfect images were present. In nearly every case such imperfections are in the nature of an elongated image, caused by a failure of the telescope to conform to the diurnal motion. A series of dashes (– – – –) indicate that the particular measure was rejected on this account.

TABULAR VALUES OF Q, m', d, AND t.

In practice, if a large number of values of an involved expression are required, the ease and rapidity with which such values can be obtained will be increased, if the functions corresponding to certain arguments are first computed and then arranged in suitable tabular form.

I have accordingly computed the values d for certain values of t and equicrescent values of Q. These quantities are arranged in tabular form. (See Table II.)

The arguments for entering this table are the measured d and the corresponding exposure time t. The corresponding

provisional magnitude m' is there found, by interpolation, in the first horizontal column of the table.

The resulting tabular magnitudes are those given by the particular DALLMEYER telescope and SEED plates No. 26, used in these investigations. For convenience and completeness, I have retained and used the quantity Q throughout the whole discussion in preference to m', as the value of Q will not be affected by any subsequent change in the light ratio which it may be found advisable to make at any future time. I have therefore also added another horizontal argument giving the values of Q.

In using the table it should be remembered that the provisional unit of brightness is that given by *Polaris* at the zenith-distance, 52° 40′, and the provisional magnitude at that zenith-distance is 2.00. These units were adopted simply as a matter of convenience, since the photographic absorption was not known until the present investigation was completed.

In order, however, to make the photographic and visual results directly comparable, *Polaris* will, as heretofore, be taken as the standard star, but the brightness (1.00) and the magnitude (2.00) finally assigned will be that which the star would have if it could be observed in the zenith of the LICK Observatory.

For facilitating the use of Table II in the finally adopted system of brightness and magnitude, I have placed the new arguments, corresponding to the tabular d, at the bottom of the page.

As will be shown farther on, the atmospheric absorption of the photographic rays at 52° 40′ zenith-distance amounts to 0.51 magnitudes on the provisional scale, consequently, the magnitude of *Polaris* in the zenith is $1^m.49$.

If, therefore, we adopt 2.00 as the photographic magnitude of *Polaris* in the zenith, we have simply to add $0^m.51$ to each of the corresponding tabular arguments for magnitude to obtain the new tabular arguments for magnitude.

To find the relation which exists between the provisional tabular values of Q, and the corresponding values of Q' in the new system, we can write the two equations:

$$Q^2 = (0.4)^{m-2} \tag{29}$$
$$Q'^2 = (0.4)^{m-2+0.51} \tag{30}$$

Passing to logarithms, and taking the difference between equations (29) and (30), we readily deduce the relation

$$Q' = 0.79\ Q \qquad (31)$$

Hence, having given the tabular value of Q corresponding to a given d in the provisional system, we have only to multiply it by 0.79 to obtain the tabular Q', corresponding to the same value of d in the new system.

In the following table I have carried the tabular quantities to extreme values of Q, d, and t, not with the expectation that the relations will be found to be strictly accurate at these extreme limits, but rather for the purpose of giving a general numerical view, so to speak, of the whole theory, and also to more easily enable others to compare their results with those here given:

TABLE II.

Provisional m'	10.49	9.61	8.99	8.51	8.11	7.77	7.48	7.23	7.00	5.49	4.61	3.99	2.48	1.61	0.98	0.50
Provisional Q	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.20	0.30	0.40	0.80	1.20	1.60	2.00
Exposure Times.		d	d	d	d	d	d	d	d	d	d	d	d	d	d	d
2^s		0.0005	0.0009	0.0012	0.0015	0.0018	0.0020	0.0021	0.0023	0.0034	0.0041	0.0046	0.0060	0.0070	0.0078	0.0085
4		.0006	.0010	.0013	.0016	.0018	.0021	.0022	.0024	.0036	.0044	.0050	.0068	.0081	.0094	.0105
8		.0006	.0010	.0013	.0017	.0019	.0021	.0023	.0025	.0038	.0047	.0054	.0076	.0093	.0110	.0125
16		.0006	.0011	.0014	.0017	.0020	.0022	.0024	.0026	.0040	.9050	.0058	.0084	.0105	.0126	.0144
32		.0006	.0011	.0014	.0018	.0020	.0023	.0024	.0027	.0042	.0053	.0062	.0092	.0117	.0141	.0164
64		.0007	.0011	.0015	.0019	.0021	.0024	.0025	.0028	.0044	.0056	.0066	.0099	.0129	.0157	.0184
128	d	.0007	.0012	.0015	.0019	.0022	.0025	.0026	.0029	.0046	.0059	.0070	.0107	.0141	.0173	.0204
256	0.0001	.0007	.0012	.0016	.0020	.0023	.0025	.0027	.0030	.0048	.0062	.0074	.0115	.0153	.0189	.0224
512	.0001	.0008	.0013	.0016	.0020	.0023	.0026	.0028	.0031	.0050	.0065	.0078	.0123	.0165	.0205	.0244
1024	.0001	.0008	.0013	.0017	.0021	.0024	.0027	.0029	.0032	.0052	.0068	.0082	.0131	.0177	.0221	0.264
2048	.0001	.0008	.0013	.0017	.0022	.0025	.0028	.0030	.0033	.0054	.0071	.0086	.0139	.0189	.0237	.0284
4096	.0001	.0009	.0014	.0018	.0022	.0025	.0029	.0031	.0034	.0056	.0074	.0090	.0147	.0201	0.253	.0303
Adopted Q	0.016	0.024	0.032	0.040	0.047	0.055	0.063	0.071	0.079	0.158	0.237	0.316	0.632	0.948	1.264	1.580
Adopted Magnitude	11.00	10.12	9.50	9.02	8.62	8.28	7.99	7.74	7.51	6.00	5.12	4.50	2.99	2.12	1.49	1.01

TABLE II—Continued.

Provisional m'	+0.10	—0.24	—0.53	—0.78	—1.01	—1.22	—1.41	—1.58	—1.74	—1.89	—2.03	—2.16	—2.29	—2.40	—2.52	—2.62
Provisional Q	2.40	2.80	3.20	3.60	4.00	4.40	4.80	5.20	5.60	6.00	6.40	6.80	7.20	7.60	8.00	8.40
Exposure Times.	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d
2s	0.0091	0.0098	0.0103	0.0109	0.0115	0.0120	0.0125	0.0130	0.0135	0.0140	0.0145	0.0150	0.0155	0.0160	0.0164	0.0169
4	.0115	.0125	.0135	.0145	.0154	.0164	.0173	.0182	.0191	.0199	.0208	.0217	.0226	.0235	.0244	.0252
8	.0139	.0153	.0167	0.181	.0194	.0207	.0220	.0234	.0246	.0259	.0272	.0285	.0298	.0311	.0323	.0336
16	.0162	.0181	.0199	.0216	.0234	.0251	.0268	.0285	.0302	.0319	.0336	.0353	.0369	.0386	.0403	.0419
32	.0186	.0209	.0231	.0252	.0274	.0295	.0316	.0337	.0358	.0379	.0399	.0441	.0441	.0462	.0482	.0503
64	.0210	.0237	.0262	.0288	.0313	.0338	.0364	.0388	.0413	.0439	.0463	.0488	.0512	.0537	.0562	.0586
128	.0234	.0265	.0294	.0324	.0353	.0382	.0411	.0440	.0469	.0498	.0526	.0555	.0584	.0613	.0641	.0670
256	.0258	.0292	.0326	.0359	.0393	.0426	.0459	.0492	.0525	.0558	.0590	.0623	.0655	.0688	.0721	.0753
512	.0282	.0320	.0358	.0395	.0433	.0469	.0507	.0543	.0581	.0617	.0654	.0691	.0727	.0764	.0800	.0836
1024	.0305	.0348	.0390	.0431	.0472	.0513	.0554	.0595	.0636	.0677	.0717	.0758	.0798	.0839	.0879	.0920
2048	.0329	.0376	.0421	.0467	.0512	.0557	.0602	.0647	.0691	.0737	.0781	.0825	.0870	.0915	.0959	.1003
4096	.0353	.0404	.0453	.0503	.0552	.0601	.0649	.0698	.0747	.0796	.0845	.0893	.0941	.0990	.1038	.1087
Adopted Q	1.896	2.212	2.528	2.844	3.160	3.476	3.792	4.108	4.424	4.740	5.056	5.372	5.688	6.004	6.320	6.636
Adopted Magnitude	+0.61	+0.27	—0.02	—0.27	—0.50	—0.71	—0.91	—1.07	—1.23	—1.38	—1.52	—1.65	—1.78	—1.89	—2.01	—2.11

ATMOSPHERIC ABSORPTION OF THE PHOTOGRAPHIC RAYS OF LIGHT.

Owing to the ever changing condition of our atmosphere, and to the want of definite information as to its density at different heights for variations in temperature and pressure, the complete solution of the problem of determining the path of a ray of star-light has not yet been accomplished.

Many different expressions have been deduced by as many different investigators for the law of atmospheric refraction, and in all of these, quantities of a more or less empirical character have been so introduced that the assumed law satisfied the observed data.

The determination of the amount of star-light lost to us during its transmission through our atmosphere seems to be a still more complicated problem. In the case of refraction, observation seems to show that the relative humidity, for instance, can be almost wholly neglected so far as its effect upon the refraction is concerned. The same may be said of other causes which do not affect refraction but which do tend to diminish the final amount of light which would otherwise be received.

It is probable that a ray of light in passing through a medium of varying density suffers not only internal refraction, but also internal reflection. The exact amount of light lost by this latter property has, so far as I am aware, never been determined.

When, in addition to these difficulties, the subject is further complicated by the introduction of that still mysterious agent, the photographic plate, as a recorder of certain data, the task of satisfactorily discussing such material from a purely theoretical standpoint is a hopeless one at the present state of our knowledge.

A consideration of these difficulties impelled me to resort to the same method of deducing an *empirical* expression which should represent the terrestrial atmospheric absorption of the photographic rays of light that I had used in deriving the preceding formula for finding the photographic magnitudes of the fixed stars.

The expressions deduced by LA PLACE, SEIDEL, MÜLLER, PICKERING, and others, for the atmospheric absorption of the visual rays, are all of a more or less empirical character.

This investigation, like the preceding, was undertaken at the suggestion of Professor HOLDEN. Four different series of observations were made, as follows:

First Series....At Mt. Hamilton, in Sept., 1889 ....U. S. N. O. Telescope.
Second Series..At Cayenne, S. A., in Dec., 1889 ....U. S. N. O. Telescope.
Third Series...At Mt. Hamilton, in July and Aug., 1890..U. S. N. O. Telescope.
Fourth Series..At Mt. Hamilton, in Nov., 1891 ....Crocker Telescope.

The exposures for the third series were kindly made for me by Professor CAMPBELL, who, at the time, was spending his vacation at the LICK Observatory. The exposures of the remaining series were made by myself.

METHOD OF OBSERVING.

As the problem under consideration is one which can be said to be of a differential character, the observations and reductions can be so arranged that only systematic errors, if any, need be involved. Such errors, so far as the effect upon the final result is concerned, can be practically eliminated in the reductions.

To accomplish this object, certain conditions must be fulfilled. Among others, care must be taken that during the time required to make a single, complete, and independent determination, there shall be no changes in—

(1) The source of light.
(2) The sensitiveness of the photographic films.
(3) The atmospheric conditions.
(4) The focal length of the telescope, as defined by the distance from the objective to the sensitive film.
(5) The development of the plates.
(6) The illumination of the plates while they are being measured.

All the above-mentioned conditions can be practically fulfilled, as will appear farther on:

First—To secure constancy in the original source of light. A bright star should be selected having a declination somewhere in the neighborhood of the latitude of the point of observation, and whose position with reference to the observer's zenith is such that during the interval between the time of the star's meridian passage and the time of its rising or setting, the exposures on this star, at various zenith-distances, are free from twilight and moonlight. No star of variable light should be employed.

Second—As no two sensitive plates, taken at random from the same box, can *a priori* be said to be of positively the same degree of sensitiveness, it is evident that the only way to secure uniformity in this respect is to make all the exposures for a given determination on the same plate; and even here we assume that the film is equally sensitive in all its parts, which is not strictly true.

Third—Observations made under abnormal atmospheric conditions (clear at one altitude, and foggy, smoky, or cloudy at other altitudes) should be rejected.

Fourth—In order to be sure of the invariability of the position of the sensitive plate, so far as its distance from the objective is concerned, the same plate-holder should be used for all the exposures of a single determination; and all the images should be near the center of the field.

Both the second and fourth conditions are fulfilled, if the same plate and plate-holder are used.

Fifth—As all the images of any given series will now be found on a single plate, the method of development will evidently be the same for all. Different plates should all be developed in precisely the same way.

Sixth—It is very essential that *all* the measures of a given series be made under the same illumination; too much stress cannot be laid upon this condition. As all the exposures will be found upon a single plate, this condition can be fulfilled by making all the measures of this plate at a single sitting. To avoid the effect of personal equation, all the measures of the present paper have been made by myself.

Method of Derivation of the Fundamental Equation.

To determine the form of the function which best represents the law of apparent decrease in brightness of a star with increasing zenith-distance, the theoretical value of Q, corresponding to each measured d for exposures of 2^s, 4^s, and 8^s, was first obtained with the aid of Table II, and the mean of the separate results adopted as the value of Q at the zenith-distance corresponding to the mean of the times of observation.

Empirical equations of conditions were then formed, each value of Q furnishing the constant or absolute term of an independent equation. The form of the function must evidently be

such that its value is at a maximum for the zenith-distance zero, and at a minimum when the zenith-distance is at a maximum.

Several of the simpler formulæ which I deduced represented the observed data quite satisfactorily for zenith-distances down to 70° or 75°; but for great zenith-distances all these first attempts were found to lack generality, the residuals near the horizon being relatively large, and of a systematic character. After several tedious trials of more complicated formulæ, in each of which nearly the whole mass of available observations was worked over, and the residuals (obtained by subtracting the empirical values from the observed) discussed by a combination of graphical and analytical results, I interpolated the formula given below, which now represents the observed data, in such a way that the sum of the squares of the residuals is less than it is for any of the other formulæ discussed.

If B_0 and B denote, respectively, the photographic magnitude of a star at the zenith-distances $\zeta = 0°$ and $\zeta = \zeta°$, and if f denotes a constant depending chiefly upon the condition of the atmosphere, the equation which best represents the observed data is of the form

$$B = B_0 \left[1 - f \tan \left(\left(\frac{\zeta}{12} \right)^2 \right) \right]^2 \qquad (32)$$

In this expression $\frac{\zeta}{12}$ is to be regarded as an abstract number, the square of which represents the number of degrees of which the trigonometrical tangent is required.

First Series of Observations for Absorption.

All the observations of this series were made with the U. S. N. O. telescope already referred to. This instrument was set up on Mount Hamilton, so as to have a clear eastern sky. As there was no covering of any kind to protect it from the wind, which is often quite strong here, much trouble was experienced from this source. Besides causing a vibration of the whole instrument, the wind very frequently stopped the driving clock, which, for this instrument, is governed by a swinging pendulum which unlocks the train of wheelwork at every half vibration; sudden variations in the speed being checked by revolving fans. This form of governor works satisfactorily so long as the resist-

ance to be overcome by the clock is uniform. If, however, for any cause, the train of wheelwork lags so that the pendulum reaches the unlocking point after the escapement shaft is in proper position, the clock at once comes to a standstill. Later on the fans were inclosed in a box, which improved matters somewhat. So far as my own experience goes the very simple and wholly satisfactory contrivance now so largely used by American instrument makers to regulate the velocity of the centrifugal or rotary pendulum governor is much to be preferred.

The first series of observations for atmospheric absorption of the photographic rays was made on α *Arietis*. I should have preferred to use a much brighter star, but this one seemed to be in the best position for observation at both great and small zenith-distances, as in other directions the view was much more limited, owing to the proximity of the Observatory buildings. I also regret that in the original program it was thought to be sufficient to carry the observations to a zenith-distance of only 75°. (Later on I secured several series of observations on α *Lyrae* to a zenith-distance of 90°.)

At each altitude exposures of 2^s, 4^s, and 8^s were made on the same plate, the telescope being slightly shifted after each exposure to avoid the overlapping of the different images. A finding view telescope attached to the tube of the photographic telescope was furnished with a suitable network of wires, so that the exposures for the different sets could be properly located.

The interval of time between the exposures at different altitudes was, on an average, less than one hour.

All the exposures were made by removing the cap covering the object-glass as quickly as possible at a given beat of the chronometer, and replacing it as quickly as possible at another given beat. A little practice enables one to make the exposure times of the proper duration with a very small percentage of error for all exposures of not less than 1^s. I have, however, deemed it best to use only the exposures which are greater than 1^s.

In the following pages the comparison between theory, as represented by equation (32), and observation, is given with sufficient detail. In each equation of condition the unknown

quantities involved are Q $(=\sqrt{B})$ and f, each equation being of the form

$$\alpha - \beta\ \varphi\ (z) = Q\ * \qquad (33)$$

which $\alpha = Q_0$; $\frac{\beta}{\alpha} = f$, or $\beta = f\ Q_0$; and $\varphi\ (z) = \tan\left[\left(\frac{\zeta}{12}\right)^2\right]$.

Explanation of the Tabular Data in Tables III–VII.

The first column gives the sidereal time corresponding to the mean of the times of exposure. The corresponding hour-angle (τ), zenith-distance (z), measured diameters (d), and resulting values of Q are given in the succeeding columns under those headings. The last column gives the residual, found by subtracting the computed theoretical value at a given zenith-distance, as given by equation (32), from the observed value at the same zenith-distance.

The values of d and Q for the separate exposures of 2^s, 4^s, and 8^s are given individually, in order that, first, all the observed data may be available for any future use, and second, to show more plainly how closely the degree of accuracy is dependent upon the measured d corresponding to a given exposure time t. The values of d are given in units of the fourth decimal place of inches.

As a test for determining the sensitiveness of the particular plate, a series of exposures on *Polaris* were also made; the results will be found in Table VIII, which also gives the provisional relative values of Q and m' for both *Polaris* and α *Arietis*. The last column gives the individual results of the difference between the magnitudes of these two stars. The abnormal condition of the results for September 6th is shown to be the same for both stars.

* These values of α and β have no relation to those given in the preceding pages.

U. S. N. O. Telescope.

L. O., Sept. 4, 1889.

α Arietis.

TABLE III.

Bar., 25in.87.
Att. Ther., 76°.
Ex., 73°.

T	τ	ζ	d 2s 4s 8s	Q	Mean Q	$o-c$
20h 18m	18h 17m	73°.1	42 54 63	0.30 0.50 0.55	0.45	+0.05
20 55	18 54	66 .0	52 62	0.45 0.55	0.50	+0.02
21 33	19 32	58 .4	50 59 66	0.50 0.60 0.60	0.57	+0.02
22 48	20 47	43 .5	48 59 68	0.45 0.60 0.65	0.57	−0.07
0 0	21 59	29 .7	56 63	0.70 0.70	0.70	+0.01
0 10	22 0	27 .8	55 65 75	0.65 0.70 0.80	0.72	+0.02
1 3	23 2	18 .9	56 63 72	0.70 0.70 0.70	0.70	−0.02

Equations of Condition.

$$\begin{aligned} \alpha - 0.76\ \beta &= 0.45 \\ \alpha - 0.58\ \beta &= 0.50 \\ \alpha - 0.44\ \beta &= 0.57 \\ \alpha - 0.23\ \beta &= 0.57 \\ \alpha - 0.11\ \beta &= 0.70 \\ \alpha - 0.09\ \beta &= 0.72 \\ \alpha - 0.04\ \beta &= 0.70 \end{aligned} \qquad (34)$$

Normal Equations.

$$\begin{aligned} 7.00\ \alpha - 2.25\ \beta &= 4.21 \\ 2.25\ \alpha - 1.18\ \beta &= 1.15 \end{aligned} \qquad (35)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 0.743 \\ \beta &= 0.443 \end{aligned} \qquad (36)$$

U. S. N. O. Telescope. *α Arietis.* Bar., $25^{in}.89$

L. O., Sept. 5, 1889. TABLE IV. Att., 71°. Ex., 71°.

T	τ	ζ	d 2^s 4^s 8^s	Q	Mean Q	$o-c$
$19^h\ 45^m$	$17^h\ 44^m$	79°.4	40 45 55	0.30 0.30 0.40	0.33	−0.12
21 1	19 0	64 .8	55 60 70	0.65 0.60 0.70	0.65	−0.06
21 47	19 46	55 .0	60 67 85	0.80 0.80 1.00	0.87	+0.05
23 28	21 27	35 .1	65 77 87	1.00 1.05 1.05	1.03	+0.06
0 43	22 42	22 .3	67 77 90	1.05 1.05 1.10	1.07	+0.05

Equations of Condition.

$$\begin{aligned} \alpha - 0.96\,\beta &= 0.33 \\ \alpha - 0.56\,\beta &= 0.65 \\ \alpha - 0.38\,\beta &= 0.87 \\ \alpha - 0.15\,\beta &= 1.03 \\ \alpha - 0.06\,\beta &= 1.07 \end{aligned} \tag{37}$$

Normal Equations.

$$\begin{aligned} 5.00\,\alpha - 2.11\,\beta &= 3.95 \\ 2.11\,\alpha - 1.59\,\beta &= 1.22 \end{aligned} \tag{38}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.057 \\ \beta &= 0.633 \end{aligned} \tag{39}$$

U. S. N. O. Telescope. *α Arietis.* Bar., $25^{in}.90$.

L. O., Sept. 6, 1889. TABLE V. Att., 71°. Ex., 71°.

T	τ	ζ	d 2^s 4^s 8^s	Q	Mean Q	$o-c$
19^h 40^m	17^h 39^m	80°.3	30 40 45	0.15 0.25 0.30	0.23	+0.04
21 3	19 2	64 .6	35 45 50	0.20 0.30 0.35	0.28	− 0.03
21 53	19 52	54 .4	40 45 55	0.30 0.30 0.40	0.33	− 0.02
23 4	21 3	40 .3	40 50 55	0.30 0.40 0 40	0.37	− 0.03
0 0	21 59	29 .7	45 55 60	0.40 0.50 0.50	0.47	+ 0.05

Equations of Condition.

$$\begin{aligned} \alpha - 0.99\ \beta &= 0.23 \\ \alpha - 0.55\ \beta &= 0.28 \\ \alpha - 0.37\ \beta &= 0.33 \\ \alpha - 0.20\ \beta &= 0.37 \\ \alpha - 0.11\ \beta &= 0.47 \end{aligned} \tag{40}$$

Normal Equations.

$$\begin{aligned} 5.00\ \alpha - 2.22\ \beta &= 1.68 \\ 2.22\ \alpha - 1.47\ \beta &= 0.62 \end{aligned} \tag{41}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 0.452 \\ \beta &= 0.261 \end{aligned} \tag{42}$$

U. S. N. O. Telescope. *α Arietis.* Bar., 25in.87.
L. O., Sept. 7, 1889. TABLE VI. Att., 72°.
Ex., 72°.

T	τ	ζ	d	Q	Mean Q	$o-c$
20h 21m	18h 20m	72°.6	50 60	0.50 0.60	0.55	0.00
21 29	19 28	59 .2	60 70 70	0.80 0.85 0.70	0.78	+0.01
22 17	20 16	49 .7	60 70 85	0.80 0.85 1.00	0.88	0.00
23 15	21 14	38 .1	65 75 85	1.00 1.00 1.00	1.00	+0.04
0 31	22 30	24 .2	65 75 90	1.00 1.00 1.10	1.03	−0.03

Equations of Condition.

$$\begin{aligned} \alpha - 0.74\,\beta &= 0.55 \\ \alpha - 0.45\,\beta &= 0.78 \\ \alpha - 0.31\,\beta &= 0.88 \\ \alpha - 0.18\,\beta &= 1.00 \\ \alpha - 0.07\,\beta &= 1.03 \end{aligned} \tag{43}$$

Normal Equations.

$$\begin{aligned} 5.00\,\alpha - 1.75\,\beta &= 4.24 \\ 1.75\,\alpha - 0.88\,\beta &= 1.28 \end{aligned} \tag{44}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.113 \\ \beta &= 0.758 \end{aligned} \tag{45}$$

U. S. N. O. Telescope. *α Arietis.* Bar., $25^{in}.83$.

L. O., Sept. 14, 1889. TABLE VII. Att., 67°. Ex., 67°.

T	τ	ζ	d 2^s 4^s 8^s	Q	Mean Q	$o-c$
$20^h\ 18^m$	$18^h\ 17^m$	73°.1	49 53 62	0.50 0.45 0.55	0.50	+0.02
20 55	18 54	66 .0	51 57 67	0.55 0.55 0.65	0.58	+0.02
21 33	19 32	58 .4	52 62 72	0.55 0.65 0.70	0.63	−0.01
22 48	20 47	43 .5	55 65 75	0.65 0.70 0.80	0.72	−0.02
0 3	22 2	29 .1	57 67 80	0.70 0.80 0.90	0.80	−0.01
0 10	22 9	27 .9	60 67 80	0.80 0.80 0.90	0.83	+0.02
1 0	22 59	19 .3	60 67 80	0.80 0.80 0.90	0.83	−0.01

Equations of Condition.

$$\left.\begin{aligned}
\alpha-0.75\,\beta&=0.50\\
\alpha-0.58\,\beta&=0.58\\
\alpha-0.44\,\beta&=0.63\\
\alpha-0.23\,\beta&=0.72\\
\alpha-0.10\,\beta&=0.80\\
\alpha-0.09\,\beta&=0.83\\
\alpha-0.04\,\beta&=0.83
\end{aligned}\right\}\qquad(46)$$

Normal Equations.

$$\left.\begin{aligned}
7.00\,\alpha-2.23\,\beta&=4.89\\
2.23\,\alpha-1.14\,\beta&=1.34
\end{aligned}\right\}\qquad(47)$$

Solution of Normals.

$$\left.\begin{aligned}
\alpha&=0.862\\
\beta&=0.510
\end{aligned}\right\}\qquad(48)$$

NOTE.—In the following table it should be remembered that for *Polaris* the Q_0 refers to a zenith-distance 52° 40′, while for *α Arietis*, it corresponds to the zenith-distance 0°. The same remark is to be applied to the comparisons with other stars.

TABLE VIII.

Polaris.			Q_0		m'		$\Delta\ m'$
Date.	*d*	*Q*	*Polaris.*	*α Arietis.*	*Polaris.*	*α Arietis.*	
1889.							
Sept. 4	60 65 75	0.80 0.85 0.90	0.85	0.74	2.54	2.65	−0.11
Sept. 5	65 75 85	1.00 1.00 1.00	1.00	1.06	2.00	1.88	+0.12
Sept. 6	55 60 65	0.50 0.60 0.60	0.57	0.45	3.22	3.74	−0.52
Sept. 7	65 75 90	1.00 1.00 1.10	1.03	1.11	1.93	1.78	+0.15
Sept. 14	60 75 90	0.53 0.66 1.10	0 76	0.86	2.59	2.33	+0.26

The mean zenith-distance of *α Arietis*, the atmospheric pressure and temperature, and the resulting values of *f*, are given in Table IX.

α Arietis.

TABLE IX.

Date.	Mean Zenith-Distance.	Pressure.	Temperature.	$f = \frac{\beta}{\alpha}$	REMARKS.
1889.					
Sept. 4	45°.3	25in.76	73°	0.60	
Sept. 5	51 .3	25 .79	71	0.60	
Sept. 6	53 .9	25 .80	71	0.58	Moon.
Sept. 7	48 .8	25 .77	72	0.68	Moon and smoke.
Sept. 14	45 .3	25 .74	67	0.59	

That the plates exposed on *α Arietis* were not all of the same degree of sensitiveness seems to be quite plainly shown by the results given in Table VIII, where the magnitude of *Polaris*, as deduced from its *uncorrected* measured images, varies all the way from $1^m.93$ on September 7th to $3^m.22$ on September 6th. That this variation is not due to errors of observation follows from the fact that the range in magnitude of *α Arietis*, using the *uncorrected* measures, is also greatest for these same dates.

The plate exposed September 6th was the least sensitive, and that exposed September 7th the most sensitive of the whole set.

A part of this difference in magnitude for different dates may of course be due to different atmospheric conditions, the air being more free from foreign matter at one time than another.

So far as the meteorological conditions of pressure and temperature are concerned, the range, as shown in Table IX, is entirely too small to account for any considerable portion of variation in the computed results; for this same reason no reliable inferences can be drawn from this series as to the effect of pressure and temperature on the absorption of the photographic rays.

A difference in the development of the plates would also cause a variation of precisely this kind, and this particular phase of the investigation will be treated more fully farther on. In this place I only wish to call attention to the fact that those plates which give results *indicating* greater sensitiveness (whether such is the actual case or not) give as a rule larger values of f than those plates which *appear* to be less sensitive. Compare, for instance, m' and the corresponding value of f for the same plate in the above tables.

If we take the mean of all the results for *α Arietis*, giving the observations of each night the same weight, we obtain the following expression for atmospheric absorption of the photographic rays expressed in brightness:

$$B = B_0\left[1 - 0.61\,\varphi(z)\right]^2 \qquad (49)$$

DISCUSSION OF THE SECOND SERIES OF OBSERVATIONS FOR ABSORPTION.

The second series of observations for absorption was made in Cayenne, South America, to which place the LICK Observatory was enabled to send an eclipse expedition through the liberality of Hon. CHARLES F. CROCKER, a Regent of our State University. The eclipse observers from this Observatory were S. W. BURNHAM and the writer. C. H. ROCKWELL, of Tarrytown, New York, also joined our party as a volunteer observer.

In addition to the regular work of the eclipse expedition it was my intention to make an extended series of observations on a large number of bright stars, for the purpose of determining their photographic magnitudes, and also to make a very complete series of observations on atmospheric absorption of the photographic rays. That this plan of work was not as completely carried out as originally intended must be attributed wholly to the extremely unfavorable condition of the weather. During our entire stay of one month clouds were never wholly absent from the sky during an entire night, and ordinarily the difference between the dry and wet bulb thermometers was only a degree or two.

I believe it rained on nearly every day of our stay in Cayenne. Clouds would at times suddenly form in the clearest sky, so that in making exposures it was very necessary for the observer to keep the closest watch for perfectly clear spaces.

Another, and even greater, source of annoyance was the constant tendency of the objective to become covered with dew. If reliable results were to be obtained it was evidently useless to make exposures with a lens (only incompletely wiped off with a dry cloth) which might fog over before the 2^s, 4^s, and 8^s exposures could be completed.

After my first night's experience I kept a large tin can, which was open at one end, near the instrument. This can was kept in a heated state during the whole time the observations were going on, by placing it in an inverted position over a burning lamp. Just before making the exposures this can was placed over the objective end of the tube, and allowed to remain there until the heated air within the can dispelled the dew. Often it was necessary to reheat the can several times before the desired effect could be produced.

As all the exposures were made east of the meridian the early observations would correspond to those made at great zenith-distances. The instrument at these times would, ordinarily, be still somewhat warm from the day temperature, and, consequently, the objective would be more apt to be wholly free from dew than would be the case later in the evening when the stars used would be at a greater altitude. The effect of dew on the objective would, for a moderately faint star, of course tend to diminish the size of the images on the photographic plate, while for very bright stars, like *β Orionis* and *Sirius*, there would also be a blurring of the image over a considerable larger area than that occupied by the normal image. On the whole, therefore, it is quite probable that the later exposures, in spite of the precautions taken, gave images which were of less diameter than would have been obtained earlier in the evening for an equal altitude.

A very striking case, illustrating this phase of the problem, is shown on a plate exposed on December 13th. From $0^h\ 37^m$ to $2^h\ 18^m$, sidereal time, the images of *β Orionis* increased according to the usual experience, but after $2^h\ 30^m$ the images of this star actually began to decrease in size, although the star had not yet reached the meridian. The peculiar appearance of the images and the blurred outline of the trail, shown after development, at once indicated that something was wrong. On referring to my note-book the words, "objective covered with dew at close of observations," cleared up the mystery. After this night's work, which was the first made use of in Cayenne, the above mentioned precautions were taken to keep the objective as free from dew as possible. The 2^s, 4^s, and 8^s exposures were always made at times when the star appeared to be at least several degrees from the nearest clouds, and, so far as the observer could judge, of normal brightness. It was often necessary to wait half an hour or more before a suitable exposure could be made. After each set of exposures the star was allowed to trail for one minute. These trails, in many instances, show the effects of passing clouds. From the foregoing statements it is evident that the results obtained for Cayenne are not as trustworthy as is desirable.

As the data given in the following tables are arranged in the same way as for the first series, no separate explanation need be given here.

U. S. N. O. Telescope. | Cayenne, Dec. 13, 1889.

α Orionis.

TABLE X.

Aneroid, $30^{in}.0$. | Dry Ex., 76°.0. | Wet, 75°.5.

T	τ	ζ	d 2^s 4^s 8^s	Q	Mean Q	$o-c$
$0^h\ 28^m$	$18^h\ 39^m$	79°.7	50 60 70	0.50 0.60 0.70	0.60	−0.02
0 51	19 2	74 .0	60 70 80	0.80 0.85 0.90	0.85	+0.07
1 8	19 19	69 .8	60 70 85	0.85 0.85 1.00	0.88	0.00
1 22	19 33	66 .3	65 75 90	1.00 1.00 1.10	1.03	+0.08
1 46	19 57	60 .4	65 75 85	1.00 1.00 1.00	1.00	−0.06
2 9*	20 20	54 .7	65 80 90	1.00 1.15 1.10	1.08	−0.06

*Object-glass covered with dew; images from here on begin to decrease in diameter (on the plate) with diminishing zenith-distance.

Equations of Condition.

$$\begin{aligned} \alpha - 0.97\ \beta &= 0.60 \\ \alpha - 0.78\ \beta &= 0.85 \\ \alpha - 0.67\ \beta &= 0.88 \\ \alpha - 0.59\ \beta &= 1.03 \\ \alpha - 0.47\ \beta &= 1.00 \\ \alpha - 0.38\ \beta &= 1.08 \end{aligned} \tag{50}$$

Normal Equations.

$$\begin{aligned} 6.00\ \alpha - 3.86\ \beta &= 5.44 \\ 3.86\ \alpha - 2.71\ \beta &= 3.30 \end{aligned} \tag{51}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.47 \\ \beta &= 0.88 \end{aligned} \tag{52}$$

Rigel.

U. S. N. O. Telescope.
Cayenne, Dec. 13, 1889.

TABLE XI.

T	τ	ζ	d 2^s 4^s 8^s	Q	Mean Q	$o-c$	REMARKS.
$0^h\ 37^m$	$19^h\ 28^m$	69°.5	90 110 135	2.30 2.20 2.30	2.27	−0.04	That the object-glass gradually became covered with dew is plainly shown on the glass negative; the images after $2^h\ 41^m$ are all blurred and very indistinct; they have, therefore, been wholly rejected.
0 57	19 48	64 .2	095 115 140	2.60 2.40 2.40	2.47	−0.03	
1 14	20 5	60 .1	95 125 145	2.60 2.80 2.60	2.67	+0.08	
1 30	20 21	56 .2	90 120 140	2.30 2.60 2.40	(2.43)	--------	
1 52	20 43	50 .9	100 130 150	3.00 3.00 2.70	2.90	−0.05	
2 18	21 9	44 .6	105 125 150	2.30 2.80 2.70	2.93	−0.05	
2 41	21 32	39 .2	105 130 160	3.30 3.00 3.00	3.10	+0.05	

Equations of Condition.

$$\begin{aligned}
\alpha - 0.66\ \beta &= 2.27\\
\alpha - 0.54\ \beta &= 2.47\\
\alpha - 0.47\ \beta &= 2.67\\
\alpha - 0.40\ \beta &= (2.43)\\
\alpha - 0.32\ \beta &= 2.90\\
\alpha - 0.24\ \beta &= 2.93\\
\alpha - 0.19\ \beta &= 3.10
\end{aligned} \qquad (53)$$

Normal Equations.

$$\begin{aligned}
6.00\ \alpha - 2.42\ \beta &= 16.34\\
2.42\ \alpha - 1.15\ \beta &= 6.31
\end{aligned} \qquad (54)$$

Solution of Normals.

$$\begin{aligned}
\alpha &= 3.37\\
\beta &= 1.61
\end{aligned} \qquad (55)$$

U. S. N. O. Telescope. *α Orionis.* Aneroid, $29^{in}.92$.

Cayenne, Dec. 15, 1889. TABLE XII. Ther. Dry, 80°.0. Wet, 76°.0.

T.	τ	ζ	d 2s 4s 8s	Q	Mean Q	$o-c$
$0^h\ 38^m$	$18^h\ 40^m$	77°.2	------ ------ 70	------ ------ 0.70	0.70	−0.06
0 56	19 7	72 .8	60 70 80	0.80 0.85 0.90	0.85	+0.01
1 31	19 42	64 .1	65 ------ 90	1.00 ------ 1.10	1.05	+0.09
2 4	20 15	55 .9	65 75 85	1.00 1.00 1.00	1.00	−0.04
2 30	20 41	49 .5	70 ------ 90	1.20 ------ 1.10	1.15	−0.05
3 3	21 14	41 .3	70 80 90	1.20 1.20 1.10	1.17	−0.01
4 4	22 15	26 .2	70 ------ 95	1.20 ------ 1.25	1.22	−0.01
5 14	23 25	9 .0	70 80 95	1.20 1.20 1.25	1.22	−0.05
5 49	0 0	2 .3	75 85 95	1.45 1.30 1.25	1.33	+0.05

Equations of Condition.

$$
\left.
\begin{aligned}
\alpha - 0.88\ \beta &= 0.70 \\
\alpha - 0.75\ \beta &= 0.85 \\
\alpha - 0.54\ \beta &= 1.05 \\
\alpha - 0.40\ \beta &= 1.00 \\
\alpha - 0.31\ \beta &= 1.15 \\
\alpha - 0.21\ \beta &= 1.17 \\
\alpha - 0.08\ \beta &= 1.22 \\
\alpha - 0.01\ \beta &= 1.22 \\
\alpha - 0.00\ \beta &= 1.33
\end{aligned}
\right\} \qquad (56)
$$

Normal Equations.

$$
\begin{aligned}
9.00\ \alpha - 3.18\ \beta &= 9.60 \\
3.18\ \alpha - 1.93\ \beta &= 2.95
\end{aligned} \qquad (57)
$$

Solution of Normals.

$$
\begin{aligned}
\alpha &= 1.28 \\
\beta &= 0.59
\end{aligned} \qquad (58)
$$

U. S. N. O. Telescope.	*Procyon.*	Aneroid, 29in.92.
Cayenne, Dec. 15, 1889.	TABLE XIII.	Ther. {Dry, 80°. Wet, 76°.

T	τ	ζ	d 2s 4s 8s	Q	Mean Q	$o-c$
1h 54m	18h 21m	84° .4	47 57	0.35 0.45	0.40	−0.19
2 12	18 39	79 .9	65 72	1.00 0.90	0.95	+0.08
2 52	19 19	69 .9	90 105	1.50 1.50	1.50	+0.17
3 27	19 54	61 .2	82 97 110	1.90 1.70 1.60	1.73	+0.14
4 10	20 37	50 .5	85 97 115	2.00 1.75 1.75	1.83	+0.01
5 40	22 7	28 .1	87 102 122	2.15 1.95 1.90	2.00	−0.16

Equations of Condition.

$$\left.\begin{aligned} \alpha - 1.17\ \beta &= 0.40 \\ \alpha - 0.98\ \beta &= 0.95 \\ \alpha - 0.67\ \beta &= 1.50 \\ \alpha - 0.49\ \beta &= 1.73 \\ \alpha - 0.32\ \beta &= 1.83 \\ \alpha - 0.10\ \beta &= 2.00 \end{aligned}\right\} \quad (59)$$

Normal Equations.

$$\begin{aligned} 6.00\,\alpha - 3.73\ \beta &= 8.41 \\ 3.73\,\alpha - 3.13\ \beta &= 4.04 \end{aligned} \quad (60)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 2.31 \\ \beta &= 1.47 \end{aligned} \quad (61)$$

U. S. N. O. Telescope. *α Orionis.*

Cayenne, Dec. 16, 1889. TABLE XIV. Aneroid, $29^{in}.90$.

T	τ	ζ	d 2^s 4^s 8^s	Q	Mean Q	$o-c$
0^h 47^m	18^h 58^m	75°.0	60 70 80	0.80 0.85 0.90	0.85	−0.02
1 23	19 34	66 .1	65	1.00	1.00	−0.01
1 51	20 2	59 .2	67 90	1.10 1.15	1.12	+0.02
2 20	20 31	52 .0	70 80 90	1.20 1.15 1.15	1.17	−0.01
3 25	21 36	35 .9	75 85 95	1.45 1.30 1.25	1.33	+0.04
5 45	23 56	2 .8	75 85	1.45 1.30	1.37	−0.03

Equations of Condition.

$$\begin{aligned} \alpha - 0.81\ \beta &= 0.85 \\ \alpha - 0.59\ \beta &= 1.00 \\ \alpha - 0.45\ \beta &= 1.12 \\ \alpha - 0.34\ \beta &= 1.17 \\ \alpha - 0.16\ \beta &= 1.33 \\ \alpha - 0.00\ \beta &= 1.37 \end{aligned} \tag{62}$$

Normal Equations.

$$\begin{aligned} 6.00\ \alpha - 2.35\ \beta &= 6.84 \\ 2.35\ \alpha - 1.36\ \beta &= 2.39 \end{aligned} \tag{63}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.40 \\ \beta &= 0.66 \end{aligned} \tag{64}$$

U. S. N. O. Telescope. *Sirius.*

Cayenne, Dec. 16, 1889. TABLE XV.

T	τ	ζ	d 2^s 4^s 8^s	Q	Mean Q	$o-c$
$1^h\ 17^m$	$18^h\ 37^m$	82°.6	87 100	1.40 1.35	1.37	−0.14
1 57	19 17	73 .1	100 125 140	2.95 2.80 2.60	2.78	+0.23
3 28	20 48	52 .1	115 140 190	4.00 3.40 3.90	3.77	−0.06
4 59	22 19	32 .9	120 200	4.40 4.20	4.30	−0.19
5 51	23 11	24 .7	125 175 220	4.80 4.90 4.80	4.83	+0.15

Equations of Condition.

$$\begin{aligned} \alpha - 1.08\,\beta &= 1.37 \\ \alpha - 0.75\,\beta &= 2.78 \\ \alpha - 0.34\,\beta &= 3.77 \\ \alpha - 0.13\,\beta &= 4.30 \\ \alpha - 0.07\,\beta &= 4.83 \end{aligned} \qquad (65)$$

Normal Equations.

$$\begin{aligned} 5.00\,\alpha - 2.73\,\beta &= 17.05 \\ 2.73\,\alpha - 1.87\,\beta &= 5.74 \end{aligned} \qquad (66)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 4.90 \\ \beta &= 3.14 \end{aligned} \qquad (67)$$

In Table XVI are given the mean values of ζ, pressure, temperature, Q_0, and f for each date and star.

TABLE XVI.

Date.	Star.	Mean Z.-D.	Pressure.	Temper-ature.	Q_0	$f=\frac{\beta}{\alpha}$
1889.						
December 13	*α Orionis.*	67°.5	30in.00	76°	1.47	0.59
December 15		44 .3	29 .92	80	1.28	0.46
December 16		48 .5	29 .90	----------	1.40	0.47
December 13	*Rigel.*	55 .0	30 .00	76	3.37	0.46
December 15	*Procyon.*	62 .3	29 .92	80	2.31	0.64
December 16	*Sirius.*	53 .1	29 .90	----------	4.90	0.64

The separate results for the value of $f=\frac{\beta}{\alpha}$ as found for Cayenne at sea-level are given in Table XVII.

TABLE XVII.

Star.	$f=\frac{\beta}{\alpha}$	Weight.
α Orionis	0.51	1
Rigel	0.46	1
Procyon	0.64	2
Sirius	0.64	4

In the column headed "weight," *α Orionis* has been given such small weight, firstly, because it is a variable star, and secondly, because its spectral type is different from the other stars, and consequently the coefficient of absorption may be different. To *Rigel* has been assigned the same weight, because it was only used on the first night, for which the conditions were rather uncertain. In the case of *Procyon* the zenith-distance was greater than for any of the other stars, while for giving good measurable images of a star near the horizon *Sirius* is by far the best source of stellar light. *Polaris* was photographed on two occasions; the data and results are given in Table XVIII.

Polaris.

Cayenne, Dec., 1889.

TABLE XVIII.

Date.	T	τ	ζ	$\varphi(\zeta)$	d 8^s 16^s 32^s 64^s	Q	Q		m'	
							Obs.	Comp.	Obs.	Comp.
Dec. 15..	h m 2 30	h m 1 12	83°.7	1.14	50 55 60 65	0.35 0.35 0.40 0.40	0.37	0.40	4.17	3.99
Dec. 17..	3 18	2 0	83 .8	1.14	55 60 65 70	0.40 0.40 0.45 0.45	0.42	0.40	3.89	3.99

To obtain the computed values of Q, it must be remembered that *Polaris* has been given the brightness 1.00, and the photographic magnitude 2.00 for a zenith-distance equal to the latitude of Mount Hamilton.

As found from both the preceding Mount Hamilton series and the Cayenne observations, the value of the factor f is very nearly 0.60. In the equation

$$Q = Q_0\Big(1 - 0.60\ \varphi(\zeta)\Big) \tag{68}$$

We must place $Q = 1$ and $\zeta = 52°\ 40'$, and solve for Q_0. For this value of ζ we have $\varphi(\zeta) = 0.35$; hence,

$$Q_0 = 1.26 \tag{69}$$

Substituting this value of Q_0 in equation (70), and placing $\varphi(\zeta) = 1.14$, we obtain the tabulated value 0.40 for Q; the corresponding magnitude is 3.99. The mean of the two observed photographic magnitudes is 4.03; the practical agreement between theory and observation is therefore all that could be desired.

Taking the weighted mean of all the determinations made at Cayenne, we have the expression

$$B = B_0\Big(1 - 0.59\ \varphi(\zeta)\Big)^2 \tag{70}$$

One would naturally expect that at sea-level the value of the factor f should come out greater than for a considerable altitude, but the figures do not show such a condition of things.

Perhaps, however, the effect of dew on the object-glass has not been completely eliminated. If a simultaneous series of observations had been carried on for *decreasing* star-altitudes, the effect of a gradual dewing of the object-glass would have been to cause an *increase* in resulting value of *f*.

In a clear sky the stars, at considerable altitudes, appeared fully as bright at Cayenne as they do on Mount Hamilton, so far as the observer could judge by estimation.

Discussion of the Third Series of Observations for Absorption.

After our return from Cayenne the U. S. N. O. telescope was again set up on Mount Hamilton, and a third series of observations undertaken. At the time I was busily engaged on "A Mechanical Theory of the Corona," so Professor Campbell kindly consented to make the exposures of this series for me while the Dallmeyer telescope was still available.

As in the first series, there chanced to be no suitable very bright star on which the exposures could be made. It was finally decided to use α *Andromedae.*

To utilize the whole time available for making suitable exposures, five different plate-holders were used. Each plate-holder was carefully fitted to the tube, so that the sensitive film in every case was at the same distance from the objective. Variations of an abnormal character in the diameters of the stellar images on the different plates could now be attributed to varying sensitiveness of these plates, as the atmospheric conditions were practically the same for all the plates exposed on any given day. The tabular data and results are arranged as in the previous observations, and therefore require no further explanation.

U. S. N. O. Telescope.
July 1, 1890.
Plate No. 1.

α Andromedae.

TABLE XIX.

Bar., 25in.95.
Att., 64°.5.
Ex., 63°.0.

T	τ	ζ	d 2s 4s 8s	Q	Mean Q	$o-c$
17h 34m	—6h 29m	78°.4	55 60 70	0.65 0.60 0.70	0.65	—0.06
17 49	6 14	76 .1	60 65 75	0.80 0.70 0.80	0.77	+0.01
17 57	6 1	74 .0	60 70 80	0.80 0.85 0.90	0.85	+0.05
18 39	5 24	66 .5	65 70 80	1.00 0.85 0.90	0.92	+0.01
19 5	4 58	61 .6	65 75 85	1.00 1.00 1.00	1.00	+0.03
19 45	4 18	54 .0	65 75 80	1.00 1.00 0.90	0.97	—0.08
20 11	3 52	48 .7	70 80 85	1.20 1.20 1.10	1.13	+0.04
20 36	3 27	43 .8	70 75 90	1.20 1.00 1.10	1.10	—0.02
20 49	3 14	41 .3	70 80 90	1.20 1.20 1.10	1.17	+0.03

Equations of Condition.

$$\alpha - 0.92\,\beta = 0.65 \qquad \alpha - 0.50\,\beta = 1.00$$
$$\alpha - 0.84\,\beta = 0.77 \qquad \alpha - 0.37\,\beta = 0.97$$
$$\alpha - 0.78\,\beta = 0.85 \qquad \alpha - 0.30\,\beta = 1.13 \qquad (71)$$
$$\alpha - 0.59\,\beta = 0.92 \qquad \alpha - 0.24\,\beta = 1.10$$
$$\alpha - 0.21\,\beta = 1.22$$

Normal Equations.

$$\begin{aligned} 9.00\,\alpha - 4.75\,\beta &= 8.56 \\ 4.75\,\alpha - 3.10\,\beta &= 4.16 \end{aligned} \qquad (72)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.273 \\ \beta &= 0.608 \end{aligned} \qquad (73)$$

U. S. N. O. Telescope.
July 1, 1890.
Plate No. 2.

α Andromedae.

TABLE XX.

Ba
Att
Ex

T	τ	ζ	d	Q	Mean Q
17h 24m	—6h 39m	80°.2	55 60 65	0.65 0.60 0.60	0.62
17 39	6 24	77 .5	60 65 70	0.80 0.70 0.70	0.77
17 44	6 19	76 .6	60 70 75	0.80 0.85 0.80	0.82
18 1	6 2	73 .6	60 70 80	0.80 0.85 0.90	0.85
18 34	5 29	67 .4	65 75 85	1.00 1.00 1.00	1.00
19 11	4 52	60 .5	65 75 85	1.00 1.00 1.00	1.00
19 41	4 22	54 .6	65 75 85	1.00 1.00 1.00	1.00
20 20	3 43	46 .9	65 75 90	1.00 1.00 1.10	1.03
20 54	3 9	40 .3	70 75 85	1.20 1.00 1.00	1.07
21 21	2 42	34 .9	65 80 90	1.00 1.20 1.10	1.10

Equations of Condition.

$\alpha - 0.99\ \beta = 0.62$ $\qquad$ $\alpha - 0.47\ \beta = 1.00$

$\alpha - 0.89\ \beta = 0.77$ $\qquad$ $\alpha - 0.38\ \beta = 1.00$

$\alpha - 0.86\ \beta = 0.82$ $\qquad$ $\alpha - 0.27\ \beta = 1.03$

$\alpha - 0.77\ \beta = 0.85$ $\qquad$ $\alpha - 0.20\ \beta = 1.07$

$\alpha - 0.62\ \beta = 1.00$ $\qquad$ $\alpha - 0.15\ \beta = 1.10$

Normal Equations.

$$10.00\,\alpha - 5.60\ \beta = 9.26$$

$$5.60\,\alpha - 3.97\ \beta = 4.78$$

Solution of Normals.

$$\alpha = 1.196$$

$$\beta = 0.483$$

U. S. N. O. Telescope.
July 2, 1890.
Plate No. 1.

α Andromedae.

TABLE XXI.

Bar., $25^{in}.90$.
Att., 64°.
Ex., 63°.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 8^m$	$-6^h\ 55^m$	83°.2	60 80	0.80 0.90	0.85	−0.03
17 29	6 34	79 .4	65 75	1.00 1.00	1.00	+0.04
18 10	5 53	71 .9	70 75 90	1.20 1.00 1.10	1.10	+0.01
19 9	4 54	60 .9	70	1.20	1.20	−0.02
20 20	3 43	46 .9	75 85 95	1.45 1.30 1.25	1.33	0.00

Equations of Condition.

$$\left.\begin{aligned} \alpha - 1.18\,\beta &= 0.85 \\ \alpha - 0.96\,\beta &= 1.00 \\ \alpha - 0.72\,\beta &= 1.10 \\ \alpha - 0.48\,\beta &= 1.20 \\ \alpha - 0.27\,\beta &= 1.33 \end{aligned}\right\} \qquad (77)$$

Normal Equations.

$$\left.\begin{aligned} 5.00\,\alpha - 3.54\,\beta &= 5.48 \\ 3.54\,\alpha - 2.97\,\beta &= 3.63 \end{aligned}\right\} \qquad (78)$$

Solution of Normals.

$$\left.\begin{aligned} \alpha &= 1.480 \\ \beta &= 0.542 \end{aligned}\right\} \qquad (79)$$

U. S. N. O. Telescope. *α Andromedae.*

July 2, 1890.

TABLE XXII.

Plate No. 2.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 12^m$	$-6^h\ 51^m$	82°.4	55	0.65	0.65	−0.04
17 32	6 31	78 .8	60 65 75	0.80 0.70 0.80	0.77	+0.01
18 14	5 49	71 .1	65 80	1.00 0.90	0.95	+0.07
19 13	4 50	60 .1	65 75 85	1.00 1.00 1.00	1.00	+0.01
20 23	3 40	46 .3	65 75 90	1.00 1.00 1.10	1.03	−0.06

Equations of Condition.

$$\begin{aligned} \alpha - 1.08\ \beta &= 0.65 \\ \alpha - 0.94\ \beta &= 0.77 \\ \alpha - 0.70\ \beta &= 0.95 \\ \alpha - 0.47\ \beta &= 1.00 \\ \alpha - 0.27\ \beta &= 1.03 \end{aligned} \tag{80}$$

Normal Equations.

$$\begin{aligned} 5.00\ \alpha - 3.46\ \beta &= 4.40 \\ 3.46\ \alpha - 2.83\ \beta &= 2.83 \end{aligned} \tag{81}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.220 \\ \beta &= 0.492 \end{aligned} \tag{82}$$

U. S. N. O. Telescope. *α Andromedae.*

July 2, 1890. TABLE XXIII. Plate No. 3.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 16^m$	$-6^h\ 47^m$	81°.6	55 65 75	0.65 0.70 0.80	0.72	−0.12
17 36	6 27	78 .1	65 75 85	1.00 1.00 1.00	1.00	+0.08
18 9	5 54	72 .1	70 80 85	1.20 1.20 1.00	1.13	+0.11
19 16	4 47	59 .5	70 80 90	1.20 1.20 1.15	1.18	+0.01
20 34	3 29	44 .2	70 80 95	1.20 1.20 1.25	1.22	−0.07

Equations of Condition.

$$\begin{aligned} \alpha - 1.04\,\beta &= 0.72 \\ \alpha - 0.91\,\beta &= 1.00 \\ \alpha - 0.73\,\beta &= 1.13 \\ \alpha - 0.46\,\beta &= 1.18 \\ \alpha - 0.24\,\beta &= 1.22 \end{aligned} \qquad (83)$$

Normal Equations.

$$\begin{aligned} 5.00\,\alpha - 3.38\,\beta &= 5.25 \\ 3.38\,\alpha - 2.71\,\beta &= 3.31 \end{aligned} \qquad (84)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.431 \\ \beta &= 0.564 \end{aligned} \qquad (85)$$

U. S. N. O. Telescope. *α Andromedae.*

July 2, 1890. TABLE XXIV. Plate No. 4.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 20^m$	$-6^h\ 43^m$	81°.0	60 70	0.80 0.85	0.82	−0.03
17 39	6 24	77 .5	65 75 80	1.00 1.00 0.90	0.97	+0.04
18 19	5 44	70 .2	65 80 90	1.00 1.15 1.10	1.08	+0.02
19 19	4 44	58 .9	70 80 90	1.20 1.20 1.10	1.17	−0.04
20 39	3 24	43 .2	75 85 100	1.45 1.30 1.35	1.37	+0.02

Equations of Condition.

$$\begin{aligned} \alpha - 1.02\,\beta &= 0.82 \\ \alpha - 0.89\,\beta &= 0.97 \\ \alpha - 0.68\,\beta &= 1.08 \\ \alpha - 0.45\,\beta &= 1.17 \\ \alpha - 0.23\,\beta &= 1.37 \end{aligned} \tag{86}$$

Normal Equations.

$$\begin{aligned} 5.00\,\alpha - 3.27\,\beta &= 5.41 \\ 3.27\,\alpha - 2.54\,\beta &= 3.28 \end{aligned} \tag{87}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.502 \\ \beta &= 0.642 \end{aligned} \tag{88}$$

U. S. N. O. Telescope. *α Andromedae.*

July 2, 1890. TABLE XXV. Plate No. 5.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 24^m$	$-6^h\ 39^m$	80°.2	60 70	0.80 0.85	0.82	−0.03
17 42	6 21	77 .0	65 80	1.00 0.90	0.95	+0.04
18 22	5 41	69 .7	65 85	1.00 1.00	1.00	0.00
19 22	4 41	58 .3	70 80 90	1.20 1.15 1.10	1.15	+0.04
20 46	3 17	41 .1	70 85 90	1.20 1.30 1.10	1.20	−0.02

Equations of Condition.

$$\begin{aligned} \alpha - 0.99\ \beta &= 0.82 \\ \alpha - 0.87\ \beta &= 0.95 \\ \alpha - 0.67\ \beta &= 1.00 \\ \alpha - 0.44\ \beta &= 1.15 \\ \alpha - 0.21\ \beta &= 1.20 \end{aligned} \tag{89}$$

Normal Equations.

$$\begin{aligned} 5.00\ \alpha - 3.18\ \beta &= 5.12 \\ 3.18\ \alpha - 2.42\ \beta &= 3.07 \end{aligned} \tag{90}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.323 \\ \beta &= 0.472 \end{aligned} \tag{91}$$

U. S. N. O. Telescope.
July 30, 1890.
Plate No. 1.

α Andromedae.

TABLE XXVI.

Bar., 25in.86.
Att., 67°.
Ex., 65°.

T	τ	ζ	d	Q	Mean Q	$o-c$
17h 31m	−6h 32m	79°.0	55 65 75	0.65 0.70 0.80	0.72	−0.08
17 47	6 23	77 .3	60 - - - - - 80	0.80 - - - - - 0.90	0.85	−0.01
18 5	5 58	72 .8	65 75	1.00 1.00	1.00	+0.07
19 21	4 42	58 .5	70 80 90	1.20 1.20 1.15	1.18	+0.07
20 52	3 11	40 .7	70 - - - - - 95	1.20 - - - - - 1.25	1.22	−0.04
21 48	2 15	29 .5	70 90 95	1.20 1.45 1.25	1.30	−0.03

Equations of Condition.

$$
\left.\begin{aligned}
\alpha - 0.94\,\beta &= 0.72\\
\alpha - 0.84\,\beta &= 0.85\\
\alpha - 0.74\,\beta &= 1.00\\
\alpha - 0.44\,\beta &= 1.18\\
\alpha - 0.21\,\beta &= 1.22\\
\alpha - 0.10\,\beta &= 1.30
\end{aligned}\right\} \quad (92)
$$

Normal Equations.

$$
\left.\begin{aligned}
6.00\,\alpha - 3.27\,\beta &= 6.27\\
3.27\,\alpha - 2.38\,\beta &= 3.04
\end{aligned}\right\} \quad (93)
$$

Solution of Normals.

$$
\left.\begin{aligned}
\alpha &= 1.387\\
\beta &= 0.628
\end{aligned}\right\} \quad (94)
$$

U. S. N. O. Telescope. *α Andromedae.*

July 30, 1890. TABLE XXVII. Plate No. 2.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 34^m$	$-6^h\ 29^m$	78°.4	60 70 75	0.80 0.85 0.80	0.82	−0.05
17 50	6 13	75 .5	65 75 80	1.00 1.00 0.90	0.97	+0.05
18 7	5 56	72 .4	60 75 85	0.80 1.00 1.00	0.93	−0.03
19 23	4 40	58 .1	70 80 90	1.20 1.20 1.10	1.17	+0.06
20 56	3 7	39 .9	70 80 95	1.20 1.20 1.25	1.22	−0.01
21 54	2 9	28 .3	70 85 95	1.20 1.30 1.25	1.25	−0.02
22 24	1 39	22 .5	70 1.00	1.20 1.35	1.27	−0.02

Equations of Condition.

$$
\begin{aligned}
\alpha - 0.92\,\beta &= 0.82\\
\alpha - 0.83\,\beta &= 0.97\\
\alpha - 0.74\,\beta &= 0.93\\
\alpha - 0.43\,\beta &= 1.17\\
\alpha - 0.19\,\beta &= 1.22\\
\alpha - 0.10\,\beta &= 1.25\\
\alpha - 0.06\,\beta &= 1.27
\end{aligned}
\tag{95}
$$

Normal Equations.

$$
\begin{aligned}
7.00\,\alpha - 3.27\,\beta &= 7.63\\
3.27\,\alpha - 2.32\,\beta &= 3.18
\end{aligned}
\tag{96}
$$

Solution of Normals.

$$
\begin{aligned}
\alpha &= 1.317\\
\beta &= 0.486
\end{aligned}
\tag{97}
$$

U. S. N. O. Telescope. *α Andromedae.*

July 30, 1890. TABLE XXVIII. Plate No. 3.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 38^m$	$-6^h\ 25^m$	77°.7	65 75 85	1.00 1.00 1.00	1.00	−0.07
17 52	6 11	75 .2	70 80 90	1.20 1.20 1.10	1.17	−0.01
18 10	5 53	71 .9	75 85 95	1.40 1.30 1.25	1.32	+0.08
19 26	4 37	57 .5	80 100	1.70 1.35	1.52	+0.10
20 59	3 4	39 .3	80 90	1.70 1.50	1.60	+0.04
21 57	2 6	27 .7	80 90	1.70 1.50	1.60	−0.02
22 27	1 36	21 .9	95 110	1.65 1.60	1.62	−0.08

Equations of Condition.

$$
\left.\begin{aligned}
\alpha - 0.90\,\beta &= 1.00\\
\alpha - 0.82\,\beta &= 1.17\\
\alpha - 0.72\,\beta &= 1.32\\
\alpha - 0.42\,\beta &= 1.52\\
\alpha - 0.19\,\beta &= 1.60\\
\alpha - 0.09\,\beta &= 1.60\\
\alpha - 0.06\,\beta &= 1.62
\end{aligned}\right\} \quad (98)
$$

Normal Equations.

$$
\left.\begin{aligned}
7.00\,\alpha - 3.20\,\beta &= 9.83\\
3.20\,\alpha - 2.23\,\beta &= 3.99
\end{aligned}\right\} \quad (99)
$$

Solution of Normals.

$$
\left.\begin{aligned}
\alpha &= 1.703\\
\beta &= 0.654
\end{aligned}\right\} \quad (100)
$$

U. S. N. O. Telescope. *α Andromedae.*
July 30, 1890. TABLE XXIX. Plate No. 4.

T	τ	ζ	d	Q	Mean Q	$o-c$
17h 41m	—6h 22m	77°.2	60 70 80	0.80 0.85 0.90	0.85	—0.07
17 54	6 9	74 .8	70 85	0.85 1.00	0.92	—0.04
18 12	5 51	71 .5	65 75 90	1.00 1.00 1.10	1.03	+0.03
19 29	4 34	56 .9	70 80 95	1.20 1.20 1.25	1.22	+0.07
21 1	3 2	38 .9	80	1.20	1.20	—0.06
22 0	2 3	27 .1	85	1.30	1.30	—0.01

Equations of Condition.

$$\begin{aligned} \alpha - 0.88\ \beta &= 0.85 \\ \alpha - 0.80\ \beta &= 0.92 \\ \alpha - 0.71\ \beta &= 1.03 \\ \alpha - 0.41\ \beta &= 1.22 \\ \alpha - 0.19\ \beta &= 1.20 \\ \alpha - 0.09\ \beta &= 1.30 \end{aligned} \qquad (101)$$

Normal Equations.

$$\begin{aligned} 6.00\ \alpha - 3.08\ \beta &= 6.52 \\ 3.08\ \alpha - 2.13\ \beta &= 3.07 \end{aligned} \qquad (102)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.359 \\ \beta &= 0.503 \end{aligned} \qquad (103)$$

U. S. N. O. Telescope. *α Andromedae.*
July 30, 1890. TABLE XXX. Plate No. 5.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 44^m$	$-6^h\ 19^m$	76°.6	60 70 80	0.80 0.85 0.90	0.85	−0.11
17 56	6 7	74 .4	65	1.00	1.00	+0.01
18 14	5 49	71 .1	70 80 90	1.20 1.20 1.10	1.17	+0.13
19 32	4 31	56 .4	70 80 95	1.20 1.20 1.25	1.22	+0.04
21 4	2 59	38 .3	70 85 95	1.20 1.30 1.25	1.25	−0.05
22 3	2 0	26 .5	75 85	1.40 1.30	1.35	0.00

Equations of Condition.

$$
\left.\begin{aligned}
\alpha - 0.86\,\beta &= 0.85\\
\alpha - 0.79\,\beta &= 1.00\\
\alpha - 0.70\,\beta &= 1.17\\
\alpha - 0.41\,\beta &= 1.22\\
\alpha - 0.18\,\beta &= 1.25\\
\alpha - 0.08\,\beta &= 1.35
\end{aligned}\right\}\qquad (104)
$$

Normal Equations.

$$
\left.\begin{aligned}
6.00\,\alpha - 3.02\,\beta &= 6.84\\
3.02\,\alpha - 2.06\,\beta &= 3.17
\end{aligned}\right\}\qquad (105)
$$

Solution of Normals.

$$
\left.\begin{aligned}
\alpha &= 1.393\\
\beta &= 0.503
\end{aligned}\right\}\qquad (106)
$$

U. S. N. O. Telescope.
August 6, 1890.
Plate No. 1.

α Andromedae.

TABLE XXXI.

Bar., 26in.04.
Att., 69°.
Ex., 68°.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 44^m$	$-6^h\ 19^m$	76°.6	60 75 85	0.80 1.00 1.00	0.93	−0.02
18 32	5 31	67 .8	70 80 90	1.20 1.20 1.10	1.17	+0.04
19 29	4 34	57 .0	70 85 95	1.20 1.30 1.35	1.25	−0.03

Equations of Condition.

$$\begin{aligned} \alpha - 0.86\ \beta &= 0.93 \\ \alpha - 0.62\ \beta &= 1.17 \\ \alpha - 0.42\ \beta &= 1.25 \end{aligned} \tag{107}$$

Normal Equations.

$$\begin{aligned} 3.00\ \alpha - 1.90\ \beta &= 3.35 \\ 1.90\ \alpha - 1.30\ \beta &= 2.05 \end{aligned} \tag{108}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.589 \\ \beta &= 0.745 \end{aligned} \tag{109}$$

U. S. N. O. Telescope.
August 6, 1890.

α Andromedae.

TABLE XXXII.

Plate No. 2.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 47^m$	$-6^h\ 16^m$	76°.0	65 70 80	1.00 0.85 0.90	0.92	-0.02
18 34	5 29	67 .4	70 80 90	1.20 1.20 1.10	1.17	$+0.07$
19 32	4 31	56 .4	70 85 90	1.20 1.30 1.10	1.20	-0.05

Equations of Condition.

$$\begin{aligned} \alpha - 0.84\,\beta &= 0.92 \\ \alpha - 0.61\,\beta &= 1.17 \\ \alpha - 0.41\,\beta &= 1.20 \end{aligned} \qquad (110)$$

Normal Equations.

$$\begin{aligned} 3.00\,\alpha - 1.86\,\beta &= 3.29 \\ 1.86\,\alpha - 1.25\,\beta &= 1.97 \end{aligned} \qquad (111)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.550 \\ \beta &= 0.731 \end{aligned} \qquad (112)$$

U. S. N. O. Telescope.
August 6, 1890.

α Andromedae.
TABLE XXXIII.

Poor focus.
Plate No. 3.

T	τ	ζ	d	Q	Mean Q	$o-c$
17h 50m	— 6h 13m	75°.5	65 70 80	1.00 0.85 0.90	0.92	+0.01
18 37	5 26	66 .8	70 80 90	1.20 1.20 1.10	1.17	+0.02
19 34	4 29	56 .0	70 80 95	1.20 1.20 1.25	1.22	−0.03

Equations of Condition.

$$\begin{aligned} \alpha - 0.83\,\beta &= 0.92 \\ \alpha - 0.60\,\beta &= 1.17 \\ \alpha - 0.40\,\beta &= 1.22 \end{aligned} \qquad (113)$$

Normal Equations.

$$\begin{aligned} 3.00\,\alpha - 1.83\,\beta &= 3.31 \\ 1.83\,\alpha - 1.21\,\beta &= 1.95 \end{aligned} \qquad (114)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.545 \\ \beta &= 0.725 \end{aligned} \qquad (115)$$

U. S. N. O. Telescope. *α Andromedae.*
August 6, 1890. TABLE XXXIV. Plate No. 4.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 52^m$	$-6^h\ 11^m$	75°.2	65 85	1.00 1.00	1.00	+0.01
18 40	5 23	66 .3	70 80 90	1.20 1.20 1.10	1.17	+0.02
19 37	4 26	55 .0	70 85 95	1.20 1.30 1.25	1.25	−0.04

Equations of Condition.

$$\begin{aligned} \alpha - 0.82\ \beta &= 1.00 \\ \alpha - 0.59\ \beta &= 1.17 \\ \alpha - 0.38\ \beta &= 1.25 \end{aligned} \qquad (116)$$

Normal Equations.

$$\begin{aligned} 3.00\ \alpha - 1.79\ \beta &= 3.42 \\ 1.79\ \alpha - 1.16\ \beta &= 1.98 \end{aligned} \qquad (117)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.538 \\ \beta &= 0.667 \end{aligned} \qquad (118)$$

U. S. N. O. Telescope. *α Andromedae.*
August 6, 1890. TABLE XXXV. Plate No. 5.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 54^m$	$-6^h\ 9^m$	$74°.8$	60 70 80	0.80 0.85 0.90	0.85	+0.01
18 42	5 21	66 .0	65 75 85	1.00 1.00 1.00	1.00	0.00
19 39	4 24	55 .0	70 80 85	1.20 1.20 1.00	1.13	−0.02

Equations of Condition.

$$\begin{aligned} \alpha - 0.80\ \beta &= 0.85 \\ \alpha - 0.58\ \beta &= 1.00 \\ \alpha - 0.38\ \beta &= 1.03 \end{aligned} \qquad (119)$$

Normal Equations.

$$\begin{aligned} 3.00\ \alpha - 1.76\ \beta &= 2.88 \\ 1.76\ \alpha - 1.12\ \beta &= 1.65 \end{aligned} \qquad (120)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.235 \\ \beta &= 0.469 \end{aligned} \qquad (121)$$

U. S. N. O. Telescope.
August 12, 1890.
Plate No. 1.

α Andromedae.

TABLE XXXVI.

Bar., 25in.85.
Att., 69°.
Ex., 67°.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 56^m$	$-6^h\ 7^m$	74°.3	65 75	0.70 0.80	0.75	−0.07
18 32	5 31	67 .8	65 75	1.00 1.00	1.00	+0.08
21 50	2 13	29 .1	95	1.25	1.25	+0.03
22 50	1 13	17 .8	70 80 95	1.20 1.20 1.25	1.22	−0.04

Equations of Condition.

$$
\begin{aligned}
\alpha - 0.79\,\beta &= 0.75 \\
\alpha - 0.62\,\beta &= 1.00 \\
\alpha - 0.10\,\beta &= 1.25 \\
\alpha - 0.04\,\beta &= 1.22
\end{aligned}
\tag{122}
$$

Normal Equations.

$$
\begin{aligned}
4.00\,\alpha - 1.55\,\beta &= 4.22 \\
1.55\,\alpha - 1.04\,\beta &= 1.38
\end{aligned}
\tag{123}
$$

Solution of Normals.

$$
\begin{aligned}
\alpha &= 1.280 \\
\beta &= 0.581
\end{aligned}
\tag{124}
$$

U. S. N. O. Telescope.
August 12, 1890.

α Andromedae.

TABLE XXXVII.

Plate No. 2.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 59^m$	$-6^h\ 4^m$	73°.6	60 70 80	0.80 0.85 0.90	0.85	−0.04
18 34	5 29	67 .4	75	1.00	1.00	+0.04
19 29	4 34	57 .0	90	1.10	1.10	−0.03
20 45	3 18	41 .6	80	1.20	1.20	+0.02
21 53	2 10	28 .5	80	1.20	1.20	−0.04
22 52	1 11	17 .5	95	1.25	1.25	−0.02

Equations of Condition.

$$\begin{aligned} \alpha - 0.77\,\beta &= 0.85 \\ \alpha - 0.62\,\beta &= 1.00 \\ \alpha - 0.42\,\beta &= 1.10 \\ \alpha - 0.21\,\beta &= 1.20 \\ \alpha - 0.10\,\beta &= 1.20 \\ \alpha - 0.03\,\beta &= 1.25 \end{aligned} \tag{125}$$

Normal Equations.

$$\begin{aligned} 6.00\,\alpha - 2.15\,\beta &= 6.60 \\ 2.15\,\alpha - 1.20\,\beta &= 2.14 \end{aligned} \tag{126}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.288 \\ \beta &= 0.525 \end{aligned} \tag{127}$$

U. S. N. O. Telescope. *α Andromedae.*

August 12, 1890. TABLE XXXVIII. Plate No. 3.

T	τ	ζ	d	Q	Mean Q	$o-c$
$18^h\ 1^m$	$-6^h\ 2^m$	73°.4	60 70 80	0.80 0.85 0.90	0.85	−0.03
18 36	5 27	67 .0	- - - - - - - - - - 85	- - - - - - - - - - 1.00	1.00	0.00
19 32	4 31	56 .4	70 - - - - - - - - - -	1.20 - - - - - - - - - -	1.20	+0.06
20 48	3 15	41 .5	- - - - - 85 - - - - -	- - - - - 1.30 - - - - -	1.30	+0.01
21 55	2 8	28 .1	75 85 95	1.45 1.30 1.25	1.33	−0.04

Equations of Condition.

$$\begin{aligned} \alpha - 0.76\ \beta &= 0.85 \\ \alpha - 0.60\ \beta &= 1.00 \\ \alpha - 0.41\ \beta &= 1.20 \\ \alpha - 0.21\ \beta &= 1.30 \\ \alpha - 0.09\ \beta &= 1.33 \end{aligned} \tag{128}$$

Normal Equations.

$$\begin{aligned} 5.00\ \alpha - 2.07\ \beta &= 5.68 \\ 2.07\ \alpha - 1.16\ \beta &= 2.13 \end{aligned} \tag{129}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.439 \\ \beta &= 0.733 \end{aligned} \tag{130}$$

U. S. N. O. Telescope.
August 12, 1890.

α Andromedae.

TABLE XXXIX.

Plate No. 4.

T	τ	ζ	d	Q	Mean Q	$o-c$
$18^h\ 4^m$	$-5^h\ 59^m$	73°.0	65 75 85	1.00 1.00 1.00	1.00	−0.04
18 39	5 24	66 .5	70 80 90	1.20 1.20 1.10	1.17	+0.04
19 35	4 28	55 .8	70 85 95	1.20 1.30 1.25	1.25	+0.02
20 51	3 12	40 .9	75 85 95	1.45 1.30 1.35	1.37	+0.03
21 58	2 5	27 .5	---------- ---------- 100	---------- ---------- 1.35	1.35	−.05

Equations of Condition.

$$
\begin{aligned}
\alpha - 0.75\ \beta &= 1.00 \\
\alpha - 0.59\ \beta &= 1.17 \\
\alpha - 0.40\ \beta &= 1.25 \\
\alpha - 0.20\ \beta &= 1.37 \\
\alpha - 0.09\ \beta &= 1.35
\end{aligned}
\qquad (131)
$$

Normal Equations.

$$
\begin{aligned}
5.00\ \alpha - 2.03\ \beta &= 6.14 \\
2.03\ \alpha - 1.12\ \beta &= 2.33
\end{aligned}
\qquad (132)
$$

Solution of Normals.

$$
\begin{aligned}
\alpha &= 1.450 \\
\beta &= 0.548
\end{aligned}
\qquad (133)
$$

U. S. N. O. Telescope. *α Andromedae.*
August 12, 1890.

TABLE XL.

Plate No. 5.

T	τ	ζ	d	Q	Mean Q	$o-c$
$18^h\ 7^m$	$-5^h\ 56^m$	72°.5	60 65 75	0.80 0.70 0.80	0.77	−0.01
18 41	5 22	66 .2	70	0.85	0.85	+0.01
20 54	3 9	40 .3	90	1.10	1.10	+0.01
22 1	2 2	26 .9	70	1.20	1.20	+0.03
23 2	1 1	15 .8	80 95	1.15 1.25	1.20	−0.01

Equations of Condition.

$$\begin{aligned} \alpha - 0.59\,\beta &= 0.77 \\ \alpha - 0.51\,\beta &= 0.85 \\ \alpha - 0.19\,\beta &= 1.10 \\ \alpha - 0.09\,\beta &= 1.20 \\ \alpha - 0.03\,\beta &= 1.20 \end{aligned} \qquad (134)$$

Normal Equations.

$$\begin{aligned} 5.00\,\alpha - 1.41\,\beta &= 5.12 \\ 1.41\,\alpha - 0.66\,\beta &= 1.24 \end{aligned} \qquad (135)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.244 \\ \beta &= 0.780 \end{aligned} \qquad (136)$$

U. S. N. O. Telescope.
August 13, 1890.
Plate No. 1.

α Andromedae.

TABLE XLI.

Bar., $25^{in}.89$.
Att., 65°.
Ex., 64°.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 28^m$	$-6^h\ 35^m$	79°.5	60 70	0.80 0.85	0.82	−0.02
18 22	5 41	69 .7	65 85	1.00 1.00	1.00	+0.01
19 45	4 18	53 .8	70 80 90	1.20 1.20 1.10	1.17	+0.04
21 0	3 3	39 .1	70 85 95	1.20 1.30 1.25	1.25	+0.03
22 29	1 34	21 .6	70 85 95	1.20 1.30 1.25	1.25	−0.03

Equations of Condition.

$$\begin{aligned} \alpha - 0.96\,\beta &= 0.82 \\ \alpha - 0.67\,\beta &= 1.00 \\ \alpha - 0.37\,\beta &= 1.17 \\ \alpha - 0.19\,\beta &= 1.25 \\ \alpha - 0.06\,\beta &= 1.25 \end{aligned} \tag{137}$$

Normal Equations.

$$\begin{aligned} 5.00\,\alpha - 2.25\,\beta &= 5.49 \\ 2.25\,\alpha - 1.55\,\beta &= 2.21 \end{aligned} \tag{138}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.314 \\ \beta &= 0.485 \end{aligned} \tag{139}$$

U. S. N. O. Telescope. *α Andromedae.*

August 13, 1890. TABLE XLII. Plate No. 2

T	τ	ζ	d 2^s 4^s 8^s	Q	Mean Q	$o-c$
$17^h\ 31^m$	$-6^h\ 32^m$	$79°.0$	60	0.80	0.80	−0.01
18 35	5 28	67 .2	65 75	1.00 1.00	1.00	+0.01
19 47	4 16	53 .4	70 80 90	1.20 1.20 1.15	1.17	+0.04
21 5	2 58	38 .1	70 80 95	1.20 1.20 1.25	1.22	0.00
22 32	1 31	21 .0	70 85 95	1.20 1.30 1.25	1.25	−0.04

Equations of Condition.

$$\begin{aligned} \alpha - 0.94\ \beta &= 0.80 \\ \alpha - 0.61\ \beta &= 1.00 \\ \alpha - 0.36\ \beta &= 1.17 \\ \alpha - 0.18\ \beta &= 1.22 \\ \alpha - 0.05\ \beta &= 1.25 \end{aligned} \tag{140}$$

Normal Equations.

$$\begin{aligned} 5.00\,\alpha - 2.14\,\beta &= 5.44 \\ 2.14\,\alpha - 1.41\,\beta &= 2.06 \end{aligned} \tag{141}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.320 \\ \beta &= 0.541 \end{aligned} \tag{142}$$

U. S. N. O. Telescope.
August 13, 1890.

α Andromedae.

TABLE XLIII.

Plate No. 3.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 33^m$	$-6^h\ 30^m$	78°.6	65	1.00	1.00	+0.05
18 37	5 26	66 .9	75 85 95	1.45 1.30 1.25	1.33	−0.03
19 50	4 13	52 .8	80 90	1.70 1.50	1.60	−0.08
21 10	2 53	37 .1	90 100 120	2.30 1.80 1.85	1.98	+0.07
22 35	1 28	20 .4	90 105 120	2.30 2.00 1.85	2.05	−0.01

Equations of Condition.

$$
\left.\begin{aligned}
\alpha - 0.93\ \beta &= 1.00 \\
\alpha - 0.60\ \beta &= 1.33 \\
\alpha - 0.35\ \beta &= 1.60 \\
\alpha - 0.17\ \beta &= 1.98 \\
\alpha - 0.05\ \beta &= 2.05
\end{aligned}\right\} \qquad (143)
$$

Normal Equations.

$$
\begin{aligned}
5.00\,\alpha - 2.10\ \beta &= 7.96 \\
2.10\,\alpha - 1.37\ \beta &= 2.7\dot{3}
\end{aligned} \qquad (144)
$$

Solution of Normals.

$$
\begin{aligned}
\alpha &= 2.121 \\
\beta &= 1.259
\end{aligned} \qquad (145)
$$

U. S. N. O. Telescope. *α Andromedae.*

August 13, 1890. TABLE XLIV. Plate No. 4.

T	τ	ζ	d	Q	Mean Q	$o-c$
$17^h\ 35^m$	$-6^h\ 28^m$	78°.2	0.65 0.60	1.00 0.60	0.80	−0.09
18 39	5 24	66 .5	0.70 0.80 0.90	1.20 1.20 1.10	1.17	+0.08
19 52	4 11	52 .6	0.75 0.80 0.95	1.40 1.20 1.25	1.28	+0.06
21 15	2 48	36 .1	0.75 0.85 0.95	1.40 1.30 1.25	1.32	−0.02
22 37	1 26	20 .0	0.75 0.85 1.00	1.40 1.30 1.35	1.35	−0.05

Equations of Condition.

$$\begin{aligned} \alpha - 0.92\ \beta &= 0.80 \\ \alpha - 0.59\ \beta &= 1.17 \\ \alpha - 0.35\ \beta &= 1.28 \\ \alpha - 0.16\ \beta &= 1.32 \\ \alpha - 0.05\ \beta &= 1.35 \end{aligned} \tag{146}$$

Normal Equations.

$$\begin{aligned} 5.00\,\alpha - 2.07\ \beta &= 5.92 \\ 2.07\,\alpha - 1.35\ \beta &= 2.16 \end{aligned} \tag{147}$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.429 \\ \beta &= 0.592 \end{aligned} \tag{148}$$

U. S. N. O. Telescope.
August 13, 1890.

α Andromedae.

TABLE XLV.

Plate No. 5.

T	τ	ζ	d 2 4 8	Q	Mean Q	$o-c$
17h 38m	−6h 25m	77°.7	60 65	0.80 0.70	0.75	0.00
18 41	5 22	66 .2	65 70 80	1.00 0.85 0.90	0.92	+0.01
19 55	4 8	51 .8	65 75 90	1.00 1.00 1.10	1.03	−0.02
21 20	2 43	35 .1	70 80 90	1.20 1.20 1.10	1.17	+0.02
22 39	1 24	19 .7	70 80 90	1.20 1.20 1.10	1.17	−0.03

Equations of Condition.

$$\begin{aligned} \alpha - 0.90\ \beta &= 0.75 \\ \alpha - 0.59\ \beta &= 0.92 \\ \alpha - 0.34\ \beta &= 1.03 \\ \alpha - 0.15\ \beta &= 1.17 \\ \alpha - 0.05\ \beta &= 1.17 \end{aligned} \qquad (149)$$

Normal Equations.

$$\begin{aligned} 5.00\ \alpha - 2.03\ \beta &= 5.04 \\ 2.03\ \alpha - 1.30\ \beta &= 1.79 \end{aligned} \qquad (150)$$

Solution of Normals.

$$\begin{aligned} \alpha &= 1.226 \\ \beta &= 0.536 \end{aligned} \qquad (151)$$

Exposures on Polaris.

In addition to the observations on *α Andromedae* each plate was exposed on *Polaris*; ordinarily one set of 2ˢ, 4ˢ, and 8ˢ exposures was made on each plate at the beginning of the night's work, and a similar set at the close. The observations and results are given in Table XLVI. The last column gives the quantity (expressed in magnitudes), found by subtracting the provisional magnitude of *α Andromedae* from that of *Polaris*. (See note to Table VIII.)

TABLE XLVI.

Polaris.					*Q*		*m'*		$\Delta m'$
Date.	Plate.	*d*		*Q*	*Polaris.*	*α Andro.*	*Polaris.*	*α Andro.*	
1890. July 1..	1		65 70 75	1.00 0.85 0.80	0.88	1.25	2.28	1.52	+ 0.76
July 1..	2	65 70 75	65 70 75	1.00 0.85 0.80	0.88	1.20	2.28	1.61	+ 0.67
July 2..	1	65 70 75	65 70 75	1.00 0.85 0.80	0.88	1.48	2.28	1.15	+ 1.13
July 2..	2	60 70 75	60 70 75	0.80 0.85 0.80	0.82	1.22	2.43	1.57	+ 0.86
July 2..	3	70 75 85	65 70 75	1.05 0.90 0.90	0.95	1.43	2.12	1.22	+ 0.90
July 2..	4	65 75 85	65 70 80	1.00 0.90 0.90	0.93	1.50	2.15	1.12	+ 1.03
July 2..	5	60 70 80	60 70 80	0.80 0.85 0.90	0.85	1.32	2.35	1.40	+ 0.95
July 30..	1	65 75 80	60 70 80	0.85 0.80 0.90	0.87	1.39	2.30	1.29	+ 1.01
July 30..	2	60 65 75	60 65 75	0.80 0.70 0.80	0.77	1.32	2.56	1.38	+ 1.18

TABLE XLVI—Continued.

Polaris. Plate.	d		Q	Q *Polaris.*	Q α *Andro.*	m′ *Polaris.*	m′ α *Andro.*	Δ m′
3	75 80 85	65 75 80	1.10 1.05 0.90	1.02	1.70	1.96	0.85	+1.11
4	65 70 75	65 70 75	1.00 0.85 0.80	0.88	1.36	2.28	1.33	+0.95
5	60 65 75	60 65 75	0.80 0.70 0.80	0.78	1.35	2.54	1.35	+1.19
1	65 75 85	70 75 85	1.05 1.00 1.00	1.02	1.59	1.96	1.00	+0.96
2	65 75 85	65 70 85	1.00 0.90 1.00	0.97	1.55	2.06	1.05	+1.01
3	65 70 80	70 70 80	1.05 0.85 0.90	0.93	1.55	2.15	1.05	+1.10
4	65 70 80	65 70 --	1.00 0.85 0.90	0.92	1.54	2.18	1.07	+1.11
5	65 70 75	65 70 75	1.00 0.85 0.80	0.88	1.43	2.28	1.22	+1.06
1	60 65 75	60 65 75	0.80 0.70 0.80	0.77	1.28	2.57	1.47	+1.10

TABLE XLVI—Continued.

Polaris.				Q		m′		Δ m′
Date.	Plate.	d	Q	Polaris.	α Andro.	Polaris.	α Andro.	
1890. Aug. 12..	2	60 65 65 65 80 80	0.80 0.70 0.80	0.77	1.29	2.57	1.45	+1.12
Aug. 12..	3	65 65 75 75 90 85	1.00 1.00 1.05	1.02	1.44	1.96	1.21	+0.75
Aug. 12..	4	65 70 70 75 80 85	1.05 1.05 0.90	1.00	1.52	2.00	1.09	+0.91
Aug. 12..	5	60 60 65 70 75 80	0.80 0.80 0.80	0.80	1.24	2.48	1.54	+0.94
Aug. 13..	1	65 70 75 75 85 85	1.05 1.00 1.00	1.02	1.31	1.96	1.41	+0.55
Aug. 13..	2	65 65 75 75 85 85	1.00 1.00 1.00	1.00	1.32	2.00	1.39	+0.61
Aug. 13..	3	75 75 85 85 90 90	1.45 1.30 1.15	1.30	2.12	1.43	0.37	+1.06
Aug. 13..	4	65 65 75 75 85 80	1.00 1.00 0.90	0.97	1.43	2.06	1.22	+0.84
Aug. 13..	5	60 60 65 70 75 80	0.80 0.80 0.80	0.80	1.23	2.48	1.55	+0.93

In Table XLVII the factor *f* is given for each plate. As the several values are consistently smaller than for the other series, I was for a time at a loss to account for this difference. A comparison of the negatives developed by W. W. C. with those developed by J. M. S. at once showed that the latter films are considerably darker than the former.

As some of the negatives of this third series are darker than others, the following test, as to whether the value of the factor *f* is a function of the degree of development of the plate, was made: All the plates of the third series were fastened side by side to a large, white, semi-transparent background (a frosted window pane);. thus arranged, slight differences of shade could at once be detected. Only two grades of density will be used

to designate the degree of opacity, light (L) and dark (D); the corresponding value of f is found in the horizontal line.

It should also be remarked that on three of the six nights of observation the moon was nearly full, so that a light development under such conditions would correspond to a considerably lighter development on a moonless night, as the diffused light resulting from the presence of the moon in the sky would, to a certain extent, fog the whole plate.

TABLE XLVII.

Date.	Plate.	Development.	Mean Zenith-Distance.	Q_0	$f=\frac{\beta}{\alpha}$	Remarks.
1890.						
July 1..	1	*D*	60.5	1.25	0.44	Moon nearly full—very windy.
July 1..	2	*D*	61.2	1.20	0.40	
July 2..	1	*L*	68.5	1.48	0.37	Full moon—very windy.
July 2..	2	*L*	67.7	1.22	0.40	
July 2..	3	*L*	67.1	1.43	0.40	
July 2..	4	*L*	66.2	1.50	0.43	
July 2..	5	*L*	65.3	1.32	0.36	
July 30..	1	*L*	59.3	1.39	0.45	Moon nearly full.
July 30..	2	*L*	53.6	1.32	0.36	
July 30..	3	*L*	53.0	1.70	0.38	
July 30..	4	*L*	57.8	1.36	0.37	
July 30..	5	*L*	57.2	1.35	0.36	
Aug. 6..	1	*D*	67.1	1.59	0.47	Moon rise near close of observations.
Aug. 6..	2	*D*	66.6	1.55	0.41	
Aug. 6..	3	*D*	66.1	1.55	0.47	
Aug. 6..	4	*D*	65.5	1.54	0.43	
Aug. 6..	5	*D*	65.3	1.43	0.52	
Aug. 12..	1	*D*	47.2	1.28	0.46	
Aug. 12..	2	*D*	47.6	1.29	0.41	
Aug. 12..	3	*D*	53.3	1.44	0.51	Windy.
Aug. 12..	4	*D*	52.7	1.52	0.44	
Aug. 12..	5	*D*	44.3	1.24	0.63	
Aug. 13..	1	*L*	52.7	1.31	0.37	
Aug. 13..	2	*L*	51.8	1.32	0.41	
Aug. 13..	3	*L*	51.2	[2.12]	(0.59)	
Aug. 13..	4	*L*	50.7	1.43	0.41	
Aug. 13..	5	*D*	50.1	1.23	0.44	

In Table XLVIII the mean results for each day, together with the meteorological factors, pressure and temperature, will be found tabulated. The variations in pressure and temperature are altogether too small to enable one to decide just what effect the purely meteorological factors have upon the ab-

sorption. Such an effect can, however, probably be considered as evanescent compared with the unknown errors arising from other sources, as, for instance, impurities in the atmosphere.

TABLE XLVIII.

Date.	Development.	Mean Zenith-Distance.	Pressure.	Temperature.	Q_o	f
July 1	*D*	60°.8	25in.11	63°	1.22	0.42
July 2	*L*	70 .0	25 .07	63	1.39	0.39
July 30	*L*	56 .2	24 .96	65	1.42	0.38
Aug. 6	*D*	66 .1	25 .09	68	1.53	0.46
Aug. 12	*D*	49 .0	24 .90	67	1.34	0.49
Aug. 13	*L*	51 .3	25 .03	64	1.32	0.41

It will be noticed that values of f, corresponding to a *D* development, are all greater than those for an *L* development. The *D* developments again are all very much less dense than my own, for which the value of f is about 0.60. I therefore conclude that the small value of f, in the case of *α Andromedae*, is to some extent at least due to the difference found in the developments of the plates. An idea of the relative blackness of the films, as well as an approximation to the absolute densities, can be obtained from the following experiment: Any three negatives developed by J. M. S., when superposed, form a good dark glass for viewing the sun without telescopic aid, while it takes five of the *D* negatives, developed by W. W. C., to cut off the same amount of light. The Lick Observatory has temporarily in its possession some Harvard College Observatory plates, of which it takes seven to make a dark glass of the same degree of opacity as described above. (Bache telescope plates of D. M. stars.)

That the value of the factor f is to a certain degree dependent upon the degree of development seems to be evident from the following considerations:

The image of a bright star grows more rapidly than does the image of a faint star. In developing, the faint star will, after its first appearance, increase but very little in size; but such is not the case for a bright star—the longer the development, the larger, as a rule, will be the size of the disk. So that for a long development the ratio of the diameters of the images of the same star for the two extremes of altitude will differ more

from unity than will be the case for a short development. It consequently seems to follow that the factor f should theoretically be greater for long developments than for light developments, agreeing apparently with actual observation.

Another cause which has a tendency to diminish the value of the factor f is improper focal adjustment of the sensitive plate. It is evident that if the images are slightly out of focus, the ratio of the diameters of the images for the extreme values will differ less from unity for the faulty focus than it will for the good focus; since the ratio of the increase in diameter to the whole diameter will be much greater for the smaller images than it will be for the larger, provided the density or blackness of the badly focused image is sufficiently great to admit of proper measurement.

In the present series, however, the focus seems to have been right, so that this explanation cannot be applied to account for the discrepancy.

If we use only those values of f which correspond to the D developments of the *α Andromedae* plates, we have the expression

$$B = B_0\left[1 - 0.46\,\varphi(z)\right]^2 \qquad (152)$$

If for determining the relative photographic magnitudes of *α Andromedae* in the zenith, and *Polaris* at the pole, we assign equal weights to all the observations, we have the relative numbers

$$\left.\begin{array}{l} \text{Uncorrected magnitude of } \textit{Polaris} \text{ at the pole,} \\ \quad = 2^{m}.21 \;\ldots\ldots\; z = 56^\circ\ 40'. \\ \text{Uncorrected magnitude of } \alpha\ \textit{Androm.} \text{ in the zenith,} \\ \quad = 1^{m}.25 \;\ldots\ldots\; z = 0^\circ\ 0'. \end{array}\right\} \quad (153)$$

Photographically, therefore, *α Andromedae* can become nearly a whole magnitude brighter than *Polaris*, as seen from the LICK Observatory, if atmospheric absorption is not allowed for.

The fact that the uncorrected magnitude of *Polaris* comes out 2.21 instead of 2.00, would also seem to indicate that the plates were under-developed, using my developments as a standard of reference. If a brighter star than *α Andromedae* had been used, more satisfactory results would doubtless have been obtained, since for a comparatively faint star the images for

short exposures are small, and at great zenith-distances more or less ill-defined, so that a very slight error in the measures produces a very large error in the deduced brightness. If the observations are to be carried to the horizon, a first magnitude star must be selected, as no impression suitable for measurement can ordinarily be obtained from fainter stars.

As has already been stated, the photographic magnitude 2.00 has been given to *Polaris* at the zenith-distance 52° 40′. At the close of this paper a new unit of brightness will be adopted, in which the correction for the atmospheric absorption of the photographic rays will be allowed for in such a way that the brightness 1.00 and the magnitude 2.00 will be assigned to *Polaris*, as it would appear in the zenith of the LICK Observatory.

DISCUSSION OF THE FOURTH SERIES OF OBSERVATIONS FOR ABSORPTION.

After I had made a preliminary discussion of the several series of observations already recorded in the preceding pages, I was very anxious to supplement these observations by another series, to see whether the law which I had found to represent the observations to 80°–85° zenith-distance would also represent observed data corresponding to 90° zenith-distance.

I was all the more impelled to make these additional observations, as there seemed to be some doubt as to whether the proper explanation had been given to account for the small value of the factor f in the case of the α *Andromedae* results.

As the DALLMEYER telescope had in the meantime been returned to Washington, it was determined to use the newly mounted CROCKER telescope, containing the WILLARD lens (refigured by BRASHEAR) already spoken of.

This instrument is in a building with a revolving dome. The pointings are made with a small telescope about two feet long, which, with the photographic telescope, is fastened to an equatorial mounting by BRASHEAR. In all the previous exposures 4 x 5 SEED plates (No. 26) were used. As the plateholders for the CROCKER telescope were all of the 8 x 10 size, SEED plates of the same size were consequently used on all the exposures of this series.

As the primary object of this series was to test the law for very great zenith-distances, a star of the first magnitude (photographic) was required to give the most reliable results. For various reasons, which need not be considered here, α *Lyrae* seemed to be the most suitable star available, although its zenith-distance at the close of evening twilight was already about 45°; this, however, was not considered to be a serious objection, as the law from the zenith down to about 80° z.-d. was already known.

To determine the correction to be applied to the measured values of d' in the case of α *Lyrae*, I made a series of exposures on this star with the CROCKER telescope. The measured diameters of the images are given in the column headed d' of the following table.

According to equation (12), taken in connection with that of (14), the required correction $(c—c')$ can now be obtained from any two values of d' corresponding to given values of t.

For obvious reasons, the errors in the individual measures will have the least effect upon the resulting values of $(c—c')$ when the difference between the two values of d' is the greatest. If, therefore, we let $t_0 = 1^s$ and $t = 512^s$, the corresponding values of d'_0 and d' being respectively 0.0099 and 0.0395, the equation (16) at once gives for Q the value

$$Q = \frac{0.0296}{0.0033 \times 2.709} = 3.3 \qquad (154)$$

With this value of Q as an argument, we now take from Table II the tabular values given in the column headed d'. The fourth column contains the individual values of $d—d' = c—c'$.

TABLE XLIX.

t	d'	d	$c—c'$
1^s	0.0099	0.0071	—0.0028
2	0.0132	0.0104	0.0028
4	0.0165	0.0137	0.0028
8	0.0195	0.0170	0.0025
16	0.0225	0.0203	0.0022
32	0.0260	0.0236	0.0024
64	0.0297	0.0268	0.0029
128	0.0327	0.0301	0.0026
256	0.0360	0.0334	0.0026
512	0.0395	0.0367	0.0028

From the above comparisons, it will be seen that the agreement between theory and observation is as close as could be expected. The quantity $c—c'$ is large, and indicates that the WILLARD lens is much more effective than the DALLMEYER. I have, however, assumed this quantity to be a constant for *α Lyrae*, as all the other plates of this series were exposed, developed, and measured in the same way.

The photographic magnitude (provisional) corresponding to $Q = 3.3$ is, according to Table II, $m = -0.597$, agreeing fairly well with the magnitude given by the standard telescope.

Explanation of the Tables L–LV.

The first three columns need no explanation, as the arrangement is the same as for the preceding tables. In the fourth column, two independent measures of the diameters of each stellar image are given. As the images are slightly elongated, these measures were made along two diameters, at right angles to each other. The fifth column gives the *corrected* mean diameter, found by subtracting 0.0027 from the mean of the measured diameters. These two columns correspond to the 2^s exposures. The same explanation applies to the columns for the 4^s, 8^s, and 16^s exposures.

In forming the equations of condition in the previous investigations, I have used the mean of the results corresponding to the 2^s, 4^s, and 8^s exposures.

In the present case I have treated each set of exposures corresponding to the same value of t separately, and for two reasons: First, the short exposures near the horizon will give more uncertain results than the longer exposures, and therefore the least weight should be given to the 2^s exposures, and the greatest weight to the 16^s exposures; second, the relation between d, Q, and t may not be exactly the same for this telescope as that found for the standard instrument.

Any marked deviation from the assumed law, for this instrument, would then become apparent through the relations, which would show that the final results are in each case functions of the time of exposure.

Crocker Telescope.

Nov. 4, 1891.

α Lyrae.

TABLE L.

Bar., $25^{in}.92$.
Att., 57°.
Ex., 57°.

T	τ	ζ	2^s		4^s		8^s		16^s	
			d	d_0	d	d_0	d	d_0	d	d_0
			0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$21^h\ 42^m$	$3^h\ 9^m$	37°.2	125	98	160	125	195	168	230	198
			125		145		195		220	
22 13	3 40	42 .9	125	95	155	123	195	165	235	203
			120		145		190		225	
23 13	4 40	53 8	125	93	150	118	190	158	225	190
			115		140		180		210	
0 13	5 40	64 .5	120	85	140	108	185	145	210	178
			105		130		180		200	
1 5	6 32	73 .0	110	75	130	95	170	133	185	155
			95		115		150		180	
1 34	7 1	77 .5	95	60	100	68	145	110	170	138
			80		90		130		160	
1 48	7 15	79 .5	90	60	100	68	145	108	160	128
			85		90		125		150	

Equations of Condition.

$\alpha - \tan\left[\left(\frac{\zeta}{12}\right)^2\right]\beta$	Values of Q corresponding to d_0.				
	2^s	4^s	8^s	16^s	
$\alpha - 0.17\ \beta =$	2.80	2.80	3.20	3.20	(155)
$\alpha - 0.22\ \beta =$	2.60	2.70	3.15	3.30	
$\alpha - 0.37\ \beta =$	2.50	2.50	2.90	3.00	
$\alpha - 0.55\ \beta =$	2.00	2.10	2.55	2.75	
$\alpha - 0.75\ \beta =$	1.45	1.60	2.20	2.25	
$\alpha - 0.89\ \beta =$	0.80	0.80	1.60	1.90	
$\alpha - 0.96\ \beta =$	0.80	0.80	1.55	1.65	

Normal Equations.

First Members.	Second Members.				
	2^s	4^s	8^s	16^s	(156)
$7.00\,\alpha - 3.91\,\beta =$ $3.91\,\alpha - 2.79\,\beta =$	12.95 5.64	13.30 5.82	17.10 8.26	18.00 8.85	

Solution of Normals.

Time.		Weight.	
2^s	$\begin{cases}\alpha = 3.32\\ \beta = 2.63\end{cases}$	1	(157)
4^s	$\begin{cases}\alpha = 3.38\\ \beta = 2.65\end{cases}$	2	(158)
8^s	$\begin{cases}\alpha = 3.64\\ \beta = 2.14\end{cases}$	4	(159)
16^s	$\begin{cases}\alpha = 3.68\\ \beta = 1.99\end{cases}$	8	(160)

Observation—Computation.

Nov. 4, 1891. TABLE LI.

ζ	$o-c$			
	2^s	4^s	8^s	16^s
37°.2	−0.07	−0.13	−0.08	−0.13
42 .9	−0.14	−0.10	−0.02	+0.06
53 .8	+0.15	+0.10	+0.05	+0.05
64 .5	+0.13	+0.18	+0.08	+0.16
73 .0	+0.10	+0.21	+0.16	+0.06
77 .5	−0.22	−0.22	−0.14	−0.01
79 .5	0.00	−0.04	−0.04	−0.12

Crocker Telescope. Nov. 6, 1891.

α Lyrae.

TABLE LII.

Bar., $25^{in}.84$. Att., 46°. Ex., 47°.

T	τ	ζ	2^s d	2^s d_0	4^s d	4^s d_0	8^s d	8^s d_0	16^s d	16^s d_0
			0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$22^h\ 44^m$	$4^h\ 11^m$	48°.5	145 140	115	180 170	148	200 195	170	235	208
23 46	5 13	59 .6	140 135	108	165 160	135	190 175	155	220	193
0 46	6 13	70 .5	135 120	100	150 145	120	175 160	140	210 195	175
1 49	7 16	79 .7	115 105	83	125 120	95	150 140	118	175 160	140
2 27	7 54	85 .2	100 85	65	95 85	67	125 110	90	125 120	95
2 34	8 1	86 .1	95 75	58	90 75	55	110 90	73	115 100	80
2 42	8 9	87 .2	85 65	48	80 70	48	100 80	63	100 90	68
2 47	8 14	87 .5	75 60	40	75 65	43	90 75	55	95 85	63

Equations of Condition.

$\alpha - \tan\left[\left(\frac{\zeta}{12}\right)^2\right]\beta$	Values of Q corresponding to d_0. 2^s	4^s	8^s	16^s	
$\alpha - 0.29\ \beta =$	4.00	3.70	3.30	3.40	(161)
$\alpha - 0.46\ \beta =$	3.60	3.20	2.85	3.10	
$\alpha - 0.69\ \beta =$	3.00	2.60	2.40	2.70	
$\alpha - 0.97\ \beta =$	1.90	1.60	1.80	1.90	
$\alpha - 1.21\ \beta =$	1.00	0.80	1.10	1.00	
$\alpha - 1.26\ \beta =$	0.70	0.50	0.75	0.75	
$\alpha - 1.32\ \beta =$	0.40	0.35	0.60	0.60	
$\alpha - 1.33\ \beta =$	0.30	0.30	0.40	0.50	

Normal Equations.

First Members.	Second Members.				
	2^s	4^s	8^s	16^s	
$8.00\,\alpha - 7.53\,\beta =$ $7.53\,\alpha - 8.27\,\beta =$	14.90 9.81	13.05 8.34	13.20 9.27	13.95 9.72	(162)

Solution of Normals.

Time.		Weight.	
2^s......	$\begin{cases} \alpha = 5.21 \\ \beta = 3.56 \end{cases}$	1	(163)
4^s......	$\begin{cases} \alpha = 4.76 \\ \beta = 3.33 \end{cases}$	2	(164)
8^s......	$\begin{cases} \alpha = 4.16 \\ \beta = 2.67 \end{cases}$	4	(165)
16^s......	$\begin{cases} \alpha = 4.46 \\ \beta = 2.89 \end{cases}$	8	(166)

Observation—Computation.

Nov. 6, 1891.

TABLE LIII.

ζ	$o-c$			
	2^s	4^s	8^s	16^s
48°.5	−0.18	−0.09	−0.09	−0.23
59 .6	−0.02	−0.03	−0.08	+0.04
70 .5	+0.20	+0.14	+0.08	+0.22
79 .7	+0.14	+0.07	+0.23	+0.23
85 .2	+0.10	+0.07	+0.17	+0.02
86 .1	+0.03	−0.06	−0.05	−0.09
87 .2	−0.06	−0.01	−0.04	−0.06
87 .5	−0.18	−0.03	−0.21	−0.13

Crocker Telescope. *α Lyrae.* Bar., 26in.03.
Att., 57°.
Nov. 8, 1891. TABLE LIV. Ex., 58°.

T	τ	ζ	2^s		4^s		8^s		16^s	
			d	d_0	d	d_0	d	d_0	d	d_0
			0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23h 7m	4h 34m	52°.6	125 115	93	145 135	113	------ ------	------	------ ------	------
1 0	6 27	72 .2	105 90	70	------ ------	------	------ ------	------	------ ------	------
1 40	7 7	78 .3	100 95	70	105 95	73	120 110	88	145 130	110
2 10	7 37	82 .7	85 75	53	90 85	60	100 95	70	120 115	90
2 24	7 51	84 .7	75 65	43	80 75	50	90 85	60	------ ------	------
2 33	8 0	85 .9	70 60	38	75 65	43	80 70	48	95 85	63
2 45	8 12	87 .5	55 50	25	60 60	33	70 65	40	75 75	48
2 52	8 19	88 .3	45 45	18	55 55	28	65 55	33	65 65	38
2 57	8 24	88 .9	------ ------	------	50 50	23	55 45	23	60 55	30
3 05	8 27½	89 .3	------ ------	------	------ ------	------	50 40	18	------ ------	------

Equations of Condition.

$\alpha - \tan\left[\left(\frac{\zeta}{12}\right)^2\right]\beta$	Values of Q corresponding to d_0.				
	2^s	4^s	8^s	16^s	
$\alpha - 0.35\,\beta =$	2.50	2.35	----------	----------	
$\alpha - 0.73\,\beta =$	1.20	----------	----------	----------	
$\alpha - 0.92\,\beta =$	1.20	0.95	1.10	1.30	
$\alpha - 1.09\,\beta =$	0.60	0.60	0.70	0.90	
$\alpha - 1.19\,\beta =$	0.35	0.40	0.50	----------	(167)
$\alpha - 1.25\,\beta =$	0.25	0.30	0.30	0.50	
$\alpha - 1.33\,\beta =$	0.10	0.20	0.20	0.30	
$\alpha - 1.39\,\beta =$	0.07	0.15	0.15	0.20	
$\alpha - 1.42\,\beta =$	----------	0.10	0.09	0.10	
$\alpha - 1.45\,\beta =$	----------	----------	0.06	----------	

Normal Equations.

2^s ------ $\begin{cases} 8.00\,\alpha - 8.25\,\beta = 6.27 \\ 8.25\,\alpha - 9.37\,\beta = 4.46 \end{cases}$ (168)

4^s ------ $\begin{cases} 8.00\,\alpha - 8.94\,\beta = 5.05 \\ 8.94\,\alpha - 10.86\,\beta = 3.78 \end{cases}$ (169)

8^s ------ $\begin{cases} 8.00\,\alpha - 10.04\,\beta = 3.10 \\ 10.04\,\alpha - 12.84\,\beta = 3.44 \end{cases}$ (170)

16^s ------ $\begin{cases} 6.00\,\alpha - 7.40\,\beta = 3.35 \\ 7.40\,\alpha - 9.32\,\beta = 3.69 \end{cases}$ (171)

Solution of Normals.

Time.		Weight.	
2^s ------	$\begin{cases} \alpha = 3.17 \\ \beta = 2.31 \end{cases}$	1	(172)
4^s ------	$\begin{cases} \alpha = 3.00 \\ \beta = 2.12 \end{cases}$	2	(173)
8^s ------	$\begin{cases} \alpha = 2.68 \\ \beta = 1.83 \end{cases}$	4	(174)
16^s ------	$\begin{cases} \alpha = 3.36 \\ \beta = 2.27 \end{cases}$	8	(175)

Observation—Computation.

Nov. 8, 1891. TABLE LV.

ζ	$o-c$			
	2^s	4^s	8^s	16^s
52°.6	+0.14	+0.09	----------	----------
72 .2	−0.28	----------	----------	----------
78 .3	+0.16	−0.10	+0.10	+0.03
82 .7	−0.05	−0.09	+0.03	+0.01
84 .7	−0.07	−0.08	0.00	----------
85 .9	−0.03	−0.05	−0.08	−0.02
87 .5	0.00	+0.02	−0.03	−0.04
88 .3	+0.11	+0.10	+0.03	0.00
88 .9	----------	+0.11	+0.02	−0.04
89 .3	----------	----------	+0.05	----------

To see whether any definite relation exists between the values of α and β, and t (or f and t) for this particular telescope, we find, by taking simple means, the following figures:

TABLE LVI.

Exposure.	α	β	f	Weight.
2^s	3.90	2.83	0.70	1
4	3.71	2.70	0.73	2
8	3.49	2.21	0.63	4
16	3.83	2.38	0.62	8

The values of f resulting from the shorter exposures are, for the reason already given, less reliable than those which correspond to the longer exposures; there is, however, an evident tendency for f to increase as the exposure time diminishes. I account for this, at least partly, as follows: The correction −0.0027 has only been deduced from observations on *α Lyrae*, which correspond to exposures of one second and more. The smallest image to which this correction has been applied and found to hold good, has the diameter 0.0099 (see Table XLIX), while the smallest image measured in the series of observations for absorption has a diameter of 0.0045, which, however, was used but once to form a single equation of condition, the other values all being greater than 0.0065. It is evident that if the constant grows sensibly smaller as the image decreases from 0.0099 to the above values, the corrected

diameters for the short exposures at great zenith-distances will be too small, and consequently will have the effect of making the factor f (and, therefore, also the absorption) greater than it actually is. It is quite probable that the constant does grow smaller within the above range, but as I had no *accurate* method of testing the law for extremely short exposures, and as there are no evidences pointing to such a conclusion in Table XLIX, I deemed it best to use a constant correction for all the measures. It is of course certain that there is a limit near which this constant becomes a variable, which *decreases* in magnitude as the exposure time diminishes. A series of exposures on *Polaris* with this same telescope, in February, 1892, made on a new series of plates, gave the correction —0.0010 for all exposures from 1^s to 256^s duration, but the value of the photographic magnitude was not affected.

If we give weights to these results, which are proportional to the exposure times, the value of the factor f becomes

$$f = 0.64 \tag{176}$$

If we take the weighted mean of the separate results of each night, as found for the 2^s, 4^s, 8^s, and 16^s exposures, we have the following expressions for Q; the corresponding values of $f = \frac{\beta}{\alpha}$, and the mean zenith-distances ζ are also tabulated:

TABLE LVII.

Date.	ζ	$Q = \alpha - \beta\, \varphi(\zeta)$	f
1891.			
November 4	61°	$= 3.60 - 2.16\, \varphi(\zeta)$	0.60
November 6	75	$= 4.46 - 2.93\, \varphi(\zeta)$	0.66
November 8	81	$= 3.12 - 2.13\, \varphi(\zeta)$	0.69

The nights on which these observations were made were not of the first class, either as to clearness or steadiness of the atmosphere. It is evident that for observations at great zenith-distances a small change in the general transparency of the air will have a great effect upon the resulting brightness in the zenith, found by means of the formula applied to data derived from observations made near the horizon. It will be noticed that the individual values of α and β are unusually large on November 6th, compared with those of the other two nights.

I am inclined to attribute this difference rather to greater sensitiveness of the particular plate used than to greater transparency of the atmosphere. The practical constancy of the factor f, even for decided variations of α and β, gives evidence that the empirical law here deduced represents the true physical law with a very fair degree of accuracy.

If we take the mean of the whole series of observations the equation for α *Lyrae* takes the form

$$Q = 3.73 - 2.53\, \varphi\,(\zeta) \qquad (177)$$

The photographic magnitude (provisional) of α *Lyrae*, deduced from exposures made at a mean zenith-distance of more than 70°, and reduced to the zenith by means of the formula, is, therefore,

$$m' = -0.85 \qquad (178)$$

If, for determining the magnitude, we reject the abnormal result of November 6th, and take the mean of the values for November 4th and November 8th, we have the expression

$$Q = 3.28 - 2.24\, \varphi\,(\zeta) \qquad (179)$$

This equation gives for α *Lyrae* the provisional magnitude

$$m' = -0.58$$

agreeing closely with the provisional magnitude found by means of the standard instrument, and also with the special determination made with the WILLARD lens.

The equation which expresses the law of atmospheric absorption, as derived from all the observations on α *Lyrae*, is, therefore,

$$B = B_0\left[1 - 0.64\, \varphi\,(\zeta)\right]^2 \qquad (180)$$

It is a rather curious fact that the resulting mean value of f comes out such that for $\zeta = 90°$ we have $Q = 0.04$, corresponding to a loss of seven magnitudes. If this result could be regarded as freed from all sources of error, it would at once follow that the difference between the apparent visual and the apparent photographic magnitudes of the same star at different altitudes is not a constant quantity; the decrease in the photographic brightness being much more rapid than that of the visual brightness for increasing zenith-distances. For,

in the horizon of the LICK Observatory, the star *α Lyrae* is still plainly visible to the naked eye; it is, therefore, among other things, highly probable that much the greater portion of the light which reaches the observer from stars near the horizon comes from the visual or non-actinic part of the spectrum, agreeing, I believe, with results obtained by other methods.

Other causes which unite to bring about the peculiar result for $\zeta = 90°$, in the case of *α Lyrae*, I believe to be the following:

First—For the shorter exposures the images impressed upon the photographic plates are too small and indefinite to afford accurate data for measurement when the zenith-distance is nearly 90°.

Second—The smaller images can be considered as equivalent to images of bright stars having exposure times less than one second; the constant correction (—0.0027) used may not be strictly accurate for the extreme measures here considered.

Third—Near the horizon the images are almost always very unsteady, so that even those actinic rays which strike the photographic plate do not produce the effect which would be caused by a similar set of rays in a perfectly steady atmosphere. For *short* exposures, therefore, the observed photographic magnitude, at great zenith-distances, will, as a rule, always come out less than the true photographic magnitude; for the same reason just the opposite result might be obtained from *long* exposures on a bright star.

FINAL RESULTS BASED UPON ALL THE OBSERVATIONS.

In combining all the observations discussed in the preceding pages, the question, *just what weight should be assigned to each series*, depends mainly for its answer upon three other questions, that refer to causes which exercise a preponderating influence upon the observed results.

First—To what zenith-distance have the exposures been carried?

Second—What were the atmospheric conditions at the times the exposures were made?

Third—How were the plates developed?

The variations of *d*, for moderate zenith-distances, are so small, that no matter how accurate the observed data may be, the law for great zenith-distances could not be deduced from

such observations, as a great number of different functions could be found which would represent such data—for moderate zenith-distances—quite accurately. Hence, other things being equal, the very greatest weight should be given to those determinations which depend upon observations made at the greatest zenith-distances.

The question as to how to treat observations made under unfavorable atmospheric conditions will depend almost entirely upon the judgment of the observer. In any case such observations should be given the smallest weight.

Had the development of the plates of the third series been carried as far as for the other plates the observations on *α Andromedae* would have been given considerable weight; but the fact of the persistently small value of f points so strongly to a difference due either to excessive development in one case, or under-development in the other case, that using the first, second, and fourth series as standards, the third series must be given smaller weight. The fact that on three of the six observing nights the moon was nearly full would also require us to diminish the weight of the third series of observations.

The following are the values of f for each series, together with corresponding weights assigned:

TABLE LVIII.

Series.	f	Weight.
First—Mt. Hamilton	0.61	2
Second—Cayenne	0.59	1
Third—Mt. Hamilton	0.46	1
Fourth—Mt. Hamilton	0.64	3

Taking the weighted mean of these four values of f, we have finally the following expression for the adopted value of the "Atmospheric Absorption of the Photographic Rays of Light:"

$$B = B_0 \left[1 - 0.60\ \varphi\,(z)\right]^2 \qquad (181)$$

Explanation of Table.

With the aid of equations (181) and (4) the final tabular quantities given in Table LIX have been computed directly for each whole degree of zenith-distance from 0° to 90°.

The first column gives the observed zenith-distance, the second column the corresponding brightness, and the third column the amount of absorption, expressed in magnitudes, for the same zenith-distance. The brightness in the zenith is placed equal to unity.

TABLE LIX.

TERRESTRIAL ATMOSPHERIC ABSORPTION.

Observed Zenith-Distance.	Photographic		Observed Zenith-Distance.	Photographic		Observed Zenith-Distance.	Photographic	
	Brightness.	Absorption.		Brightness.	Absorption.		Brightness.	Absorption.
°	B	m	°	B	m	°	B	m
0	1.00	0.00	30	0.87	0.15	60	0.52	0.71
1	1.00	0.00	31	0.86	0.16	61	0.50	0.74
2	1.00	0.00	32	0.86	0.17	62	0.49	0.78
3	1.00	0.00	33	0.85	0.18	63	0.47	0.81
4	1.00	0.00	34	0.84	0.19	64	0.46	0.85
5	1.00	0.00	35	0.83	0.20	65	0.44	0.89
6	0.99	0.01	36	0.82	0.21	66	0.42	0.93
7	0.99	0.01	37	0.81	0.23	67	0.41	0.98
8	0.99	0.01	38	0.80	0.24	68	0.39	1.03
9	0.99	0.01	39	0.79	0.26	69	0.37	1.07
10	0.99	0.01	40	0.78	0.27	70	0.36	1.12
11	0.99	0.02	41	0.77	0.29	71	0.34	1.18
12	0.98	0.02	42	0.76	0.30	72	0.32	1.24
13	0.98	0.02	43	0.74	0.32	73	0.30	1.31
14	0.97	0.03	44	0.73	0.34	74	0.28	1.39
15	0.97	0.03	45	0.72	0.35	75	0.26	1.45
16	0.96	0.04	46	0.71	0.37	76	0.25	1.52
17	0.96	0.04	47	0.70	0.39	77	0.23	1.62
18	0.95	0.05	48	0.69	0.41	78	0.21	1.71
19	0.95	0.05	49	0.67	0.43	79	0.19	1.81
20	0.94	0.06	50	0.66	0.45	80	0.17	1.93
21	0.94	0.07	51	0.65	0.47	81	0.15	2.05
22	0.93	0.07	52	0.64	0.49	82	0.13	2.19
23	0.93	0.08	53	0.62	0.52	83	0.11	2.36
24	0.92	0.09	54	0.61	0.54	84	0.10	2.54
25	0.91	0.10	55	0.59	0.57	85	0.08	2.75
26	0.90	0.11	56	0.58	0.59	86	0.06	3.00
27	0.90	0.12	57	0.56	0.62	87	0.05	3.30
28	0.88	0.13	58	0.55	0.65	88	0.03	3.70
29	0.88	0.14	59	0.53	0.68	89	0.02	4.18
30	0.87	0.15	60	0.52	0.71	90	0.01	4.96

On the Probable Error of a Photographic Magnitude.

If we differentiate equation (4), regarding m' and Q as variables, we have

$$d\,m' = -2\,\frac{d\,Q}{Q} \qquad (182)$$

The expression shows that slightly erroneous values of Q, due to constant or accidental errors of observation, will produce large errors in the resulting values of m' when Q is very small. *For a given value of* t, the error of m' varies nearly inversely as Q.

In general the smaller the value of the measured d, the greater will be the error of the resulting value of m'.

However, when t is very great, other sources of error (due to (1) imperfect pointing of the telescope, (2) change in the differential refraction, (3) change in atmospheric absorption, (4) internal reflections in the photographic plate, etc.) tend to diminish the accuracy of the measured results.

The value of the probable error, *corresponding to a brightness in the neighborhood of that assigned to the standard star*, can be obtained from the data given in Table XLVI. Using all the observations of this table, I find that the probable error of an observed magnitude, deduced from the mean of any three exposures of 2^s, 4^s, and 8^s duration, comes out somewhat less than one tenth of a magnitude.

New Units of Brightness and Magnitude.

In the preceding discussion, *Polaris* has been given the photographic brightness 1.00, and magnitude 2.00, at the zenith-distance 52° 40′. From Table LIX we learn that the corresponding brightness and magnitude of the same star, if it were situated in the zenith, would be $B = 1.59$, and $m' = 1.49$. Consequently, if we change this unit of brightness, so as to make it 1.00 in the *zenith*, we have only to proceed in the way already outlined on page 11, and in equations (29), (30), and (31) to obtain the new arguments given for Table II, found at the bottom of the page.

There seems to be no marked change in the law for increasing altitudes of the observer, if the Cayenne results can be

taken as a test. Just what the absolute atmospheric absorption is in the zenith is not known, nor is this a necessary datum for the purposes of this paper, as the law has been so determined that only the observed brightness enters into the discussion.

In Table LX will be found the final photographic magnitudes of the stars mentioned in this memoir, as determined from the observations.

TABLE LX.

Star.	Place of Observation.	Provisional Photographic Magnitude (Observed).	Final Photographic Magnitude (Observed).
α Arietis	Mt. Hamilton	+2.10	+2.61
α Andromedae	Mt. Hamilton	+1.04	+1.55
α Orionis	Cayenne	(+1.30)	(+1.81)
β Orionis	Cayenne	(−0.64)	(−0.13)
α Canis Majoris	Cayenne	(−1.45)	(−0.94)
α Canis Minoris	Cayenne	(+0.10)	(+0.51)
α Lyrae	Mt. Hamilton	−0.59	−0.08
Polaris	Cayenne	(+1.52)	(+2.03)

The bracketed figures in the above table indicate that the values may be considerably in error, as no impressions of the standard star that were worthy of being used could be obtained at Cayenne; the zenith-distance of *Polaris*, even at upper culmination, being greater than 83°.

Conclusion.

In the practical application of the finally adopted values of the absorption at different zenith-distances, the character of the particular kind of plate used and the spectral type of the star observed must be taken into consideration.

A plate exposed to the action of two different sources of light of equal intensity (visually), but coming from different parts of the spectrum, will not, as a rule, be affected in the same way, for equal exposure times, by these two lights. Consequently, the law of absorption of the photographic rays, as determined with a particular kind of plate, may be different for stars having different types of spectrum.

As some kinds of plates are more sensitive to a particular part of the spectrum than others, it follows that the character of the plate may largely influence the final result obtained for absorption.

When we take into consideration the fact that the rays from the blue end of the spectrum appear to be more absorbed near the horizon than the rays from the red end, the force of the above remarks becomes still more apparent.

The results given in this paper are derived from exposures made on SEED plates, Sensitometer No. 26, of their scale, and should represent the actual conditions for a normal state of the atmosphere. For an unusually *thick* sky, suitable for observations of a certain class, it is probable that the value of the factor f is greater than 0.60; while for an unusually clear sky, its value may be somewhat less than the normal value given. It is, of course, evident that within a few degrees of the horizon considerable uncertainty will always exist in all measured data, and consequently also in the computed theoretical results.

MOUNT HAMILTON, October, 1892.

WORKS ISSUED BY THE LICK OBSERVATORY.

*** It is intended to issue, at irregular intervals, two series of works, the first, in quarto, to be known as *Publications* of the Lick Observatory; the second, in octavo, to be known as *Contributions* from the Lick Observatory. Occasional pamphlets, such as No. 2 below, may not be included in either series. At the end of every book a list of all the works issued will be given, for the convenience of librarians and others.

For the sake of uniformity, Nos. 3 and 4 below will be counted as *Contributions* Nos. 1 and 2.

1. *Publications* of the Lick Observatory of the University of California, prepared under the direction of the Lick Trustees by EDWARD S. HOLDEN. Volume I, 1887. Sacramento, 1887. 4to. [Containing a brief history of the Observatory, with descriptions of the buildings and instruments; observations of double stars by S. W. BURNHAM, 1879, of the transit of *Mercury*, 1881, by Messrs. FLOYD, HOLDEN, and BURNHAM, of the transit of *Venus*, 1882, by D. P. TODD; meteorological observations, by T. E. FRASER, 1880–85; and Reduction Tables for Mt. Hamilton, by G. C. COMSTOCK.]

2. Suggestions for Observing the Total Eclipse of the Sun on January 1, 1889, by EDWARD S. HOLDEN. Printed by authority of the Regents of the University of California. Sacramento, 1888. 8vo. [Out of print.]

3. *Contributions* from the Lick Observatory, No. 1. Reports on the Observations of the Total Eclipse of the Sun of January 1, 1889, published by the Lick Observatory. Printed by authority of the Regents of the University of California. Sacramento, 1889. 8vo. [Out of print.]

4. *Contributions* from the Lick Observatory, No. 2. Reports on the Observations of the Total Eclipse of the Sun, December 21–22, 1889, and of the Total Eclipse of the Moon, July 22, 1888, to which is added a Catalogue of the Library, published by the Lick Observatory. Printed by authority of the Regents of the University of California. Sacramento, 1891. 8vo. [Out of print.]

5. *Contributions* from the Lick Observatory, No. 3. Terrestrial Atmospheric Absorption of the Photographic Rays of Light, by J. M. SCHAEBERLE, Astronomer in the Lick Observatory. Printed by authority of the Regents of the University of California. Sacramento, 1893. 8vo.

6. *Publications* of the Lick Observatory of the University of California. Printed by authority of the Regents of the University. Volume II, 1893. Sacramento, 1893. 4to. [Containing double star observations made with the thirty-six-inch and twelve-inch refractors of the Lick Observatory from August, 1888, to June, 1892, by S. W. BURNHAM.]